ANALYSIS OF UNSTEADY HYDROMAGNETIC COUETTE FLOW WITH MAGNETIC FIELD LINES FIXED RELATIVE TO THE MOVING UPPER PLATE

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A thesis submitted in partial fulfillment for the degree of Master of Science in Applied Mathematics in the Jomo Kenyatta University of Agriculture and Technology

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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This thesis has been submitted for examination with our approval as University Supervisors.

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DEDICATION

To my parents, my brother Joseph Odhiambo, my brothers and sisters for their love, encouragement, support and commitment when I was pursuing this research.

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NOMENCLATURE

Symbol	Meaning
B	Magnetic field strength vector, [wbm^{-2}]
H_o	Magnetic flux identity along the y- axis [wbm^{-2}]
g	Acceleration due to gravity vector, [ms^{-2}]
H	Magnetic field intensity vector in Amperes per meter, [Am^{-1}]
J	Current density, [AM^{-2}]
e	Unit electric charge, [C]
E	Electric field, [v]
S	Suction/ Injection
M	Magnetic parameter
Xi	Permeability parameter
Re	Reynolds number
P	Pressure force, [nm^{-2}]
P^*	Dimensionless pressure force.
q	Velocity vector, [ms^{-1}]
$i\ , j\ , k$	Unit vector is the x, y, z directions respectively
$u\ , v\ , w$	Component of velocity vector q, [ms^{-1}]
u^*, v^*, w^*	Dimensionless velocity components
x^*, y^*, z^*	Dimensionless Cartesian co-ordinates

x , y , z	Dimensional Cartesian co-ordinates
F_i	Body forces tensor, [N]
U_i	Velocity tensor, [ms^{-1}]
x_j	Space tensor, [m]
K_p	Darcy permeability, [m^2]
D	Displacement current density, [cm^{-2}]
F_e	Electromagnetic force, [$Kgms^{-2}$]
Pr	Prandtl number
Ha	Hartman number
V_o	Suction velocity, [ms^{-1}]
u_o	Constant velocity of the moving plate
t	Time
n	Number
Greek symbol	**Meaning**
ϵ^+	Porosity
υ	Kinematic viscosity
ρ	Fluid density, [kgm^{-3}]
μ	Coefficient of viscosity, [$kgm^{-1}s$]
ϕ	Viscous dissipation functions, [s-1]

σ	Electrical conductivity, [Ω-^{1} m-^{1}]
σ_{ij}	Total stress tensor, [Nm^{-2}]
T_{ii}	Viscous Stress tensor, [Nm^{-2}]
μ_e	Magnetic permeability, [Hm^{-1}]
$\frac{D}{Dt}$	Material derivative $\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)$
∇	Gradient operator $\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{dz}\right)$
∇^2	Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right)$

LIST OF ABBREVIATIONS

FD	Finite difference
MHD	Magnetohydrodynamics
PDEs	Partial differential equations
MATLAB	Matrix Laboratory

ABSTRACT

In the present study, an unsteady magnetohydrodynamic viscous incompressible electrically conducting fluid flow between two parallel porous plates of infinite length in x and z directions subjected to a constant pressure gradient in the presence of a uniform transverse magnetic field applied parallel to the y axis with the upper plate moving with a time dependent velocity in the x direction. The lower plate is fixed while fluid suction/injection takes place through the walls of the channel with a constant velocity for suction and injection has been investigated.

The nonlinear partial differential equation governing the flow are solved numerically using the finite difference method and implemented in MATLAB. The results obtained are presented in graphs. The velocity profiles, the effect of pressure gradient, magnetic field, time and suction /injection on the flow and the effects of varying the various parameters on the velocity profile are discussed. A change on the parameters is observed to either increase, decrease or to have no effect on the velocity profile.

The MHD flow between porous plates studied in this work has many important applications in areas such as the designing of cooling systems with liquid metals, geothermal reservoirs, in petroleum and mineral industries, in underground energy transport, accelerators, MHD generators, pumps, flow meters, purification of crude oil, polymer technology and in controlling boundary layer flow over aircraft wings by injection or suction of fluid out of or into the wing among many other areas.

CHAPTER ONE

INTRODUCTION AND LITERATURE REVIEW

1.0 Introduction

Fluid is a substance that undergoes continuous deformation when acted upon by an external force. Fluids are classified as liquids or gases which are made up of molecules held together by intermolecular forces such that the fluid possesses volume but no definite shape. The molecules in a liquid are close together compared to the molecules in a gas which are always in haphazard motion in all directions colliding with one another.

Fluid flows can be described based on their dependence on time as either steady or unsteady. For steady flows, all fluid flow variables for example velocity, temperature, pressure, and density are independent on time while unsteady fluid flows are fluid flows in which the fluid properties are dependent on time i.e. the conditions change with time. In practice there are always slight variations in velocity and pressure, but if the average values are constant, the flow is considered steady.

1.1 Magnetohydrodynamics (MHD)

The word magneto hydrodynamics is composed of the words magneto (meaning magnetic), hydro (meaning liquid) and dynamics (referring to the change or progress within a system by forces). Magnetohydrodynamics is the science in which there is interaction between hydrodynamics and electromagnetism. Electromagnetism is the study of interaction between electric and magnetic fields whereas Hydrodynamics is the science that is concerned with the flow of fluids.

Electromagnetic studies involves the study of dynamics of substances that have mass and exists as solid ,liquid ,gas, or plasma moving in an electromagnetic field, especially where currents established in the matter by induction modify the field, so that the field and dynamics equations are coupled. When a conducting fluid or an ionized gas (plasma) becomes strongly magnetized when flowing in the presence of a magnetic field referred to as a magnetic fluid, an electric field is generated and electric current is also generated perpendicular to the magnetic field. The interaction of the current with the magnetic field changes the motion of the fluid and produces an induced magnetic field.

1.2 Couette Flow

The flow of a viscous fluid in the space between two parallel plates, one of which is moving relative to the other, where the flow is driven by the presence of a viscous drag acting on the fluid and the applied pressure gradient parallel to the plates is referred to as Couette flow .This flow is named in honor of Maurice Marie Couette, professor of Physics at the French University of Angers who first studied the flow.

Couette flow is frequently used to illustrate shear driven motion in which the fluid flow is induced due to the movement of one of the plates of the channel. It is investigated due to its varied application in fluid engineering, geophysics and astrophysics. The theory of Couette flow can be used for the measurement of viscosity and estimating drag forces in many applications.

1.3 Dimensional Analysis

Dimensional analysis is the process of expressing the units of any given physical quantity in terms of the fundamental units which include the units of time, mass and length. It is built on the principle of dimensional homogeneity that states that an equation expressing a physical relationship between quantities must be dimensionally homogeneous and proves to be a powerful tool in formulating problems that defy analytical solution and must be solved experimentally.

A porous medium or a porous material is a material containing pores (voids), or spaces between solid material through which liquid or gas can pass. Porosity or void fraction is a measure of the void spaces in a material, and is a fraction of the volume of voids over the total volume. Fluid flow through porous media is a subject of most common interest and has emerged as separate field of study due to the effects of porosity on fluid flow.

The study of fluid flow between porous walls is very important and is an idealization of the flow behavior that occurs in the real world in different geometries and finds varied applications in industries, engineering and in many other scientific fields.

1.4 Literature Review

Faradays (1831) discovery of induction where he concluded that electromagnetic induction is the production of an electromotive force across a conductor when it is exposed to a varying magnetic field referred to as induction. The flow of an electrically conducting fluid such as mercury under a magnetic field gives rise to an induced electric current. This discovery led to more and vast studies in the field of MHD. The concept of MHD was also studied by Hartman (1938) when he studied the effects of a conductor in an electrically conducting fluid. This followed.

Alfven (1942) did a lot of contribution in MHD where he established transverse waves in electrically conducting fluid and explained many astrophysical phenomena in relation to transverse waves. The interaction between electromagnetics and hydrodynamics was shown to be significant if the non-dimensional number $\left(\sqrt{BL(\sigma\mu_e / \rho)} / 2\right) > 1$ Linguistics (1952) where B is the magnetic field, L-characteristic length, σ -electrical conductivity, μ_e is the magnetic permeability and ρ is the density of the fluid.

A flow in a channel of a hydromagnetic fluid in which the motion of the fluid is due to movement of one of the plates of the channel, is called MHD Couette flow. MHD flows are characterized by a basic phenomenon which is the tendency of magnetic field to suppress vorticity that is perpendicular to itself which is in opposite to the tendency of viscosity to promote vorticity.

MHD Couette flow is studied by a number of researchers due its varied and wide applications in the areas of geophysics, astrophysics and fluid engineering.

Researchers have studied unsteady channel or duct flows of a viscous and incompressible fluid with or without magnetic field analyzing different aspects of the problem.

Tao (1960) studied the Magnetohydrodynamic effects on the formation of Couette flow and Katagiri (1962) investigated unsteady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid under the influence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel.

Muhuri (1963) considered this fluid flow problem within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion of one of the plates of the channel. Soundalgekar (1967) investigated unsteady MHD Couette flow of a viscous, incompressible and electrically conducting fluid near an accelerated plate of the channel under transverse magnetic field. The effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion of one of the plates of the channel, studied by Muhuri (1963). The work by Muhuri (1969) was later analyzed by Govindrajulu (1969). Mishra and Muduli (1980) discussed effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion when one of the plates starts moving with a time dependent velocity. In the above mentioned investigations, magnetic field is fixed relative to the fluid.

Singh and Kumar (1983) studied MHD Couette flow of a viscous, incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field when fluid flow within the channel is induced due to time dependent movement of one of the plates of the channel and magnetic field is fixed relative to moving plate. Singh and Kumar (1983) considered two particular cases of interest in their study viz. (i) impulsive movement of one of the plates of the channel and (ii) uniformly accelerated movement of one of the plates of the channel and concluded that the magnetic field tends to accelerate fluid velocity when there is impulsive movement of one of the plates of the channel and when there is uniformly accelerated movement of one of the plates of the channel.

Katagiri (1962) studied the problem when the flow was induced due to impulsive motion of one of the plates while Muhuri (1963) studied the problem with accelerated motion of one of the plates. Both had considered that the magnetic lines of force are fixed relative to the fluid. Singh and Kumar (1983) considered the problem studied by Katagiri (1962) and Muhuri (1963) in a non-porous channel with the magnetic lines of force fixed relative to the moving plate. Khan *et al.* (2006) investigated MHD flow of a generalized Oldroyd-B fluid in a porous space taking Hall current into account while Khan *et al.* (2007), considered MHD transient flows of an Oldroyd-B fluid in a channel of rectangular cross-section in a porous medium. The influence of Hall current and heat transfer on the steady MHD flow of a generalized Burgers' fluid between two eccentric rotating infinite discs of different temperatures was studied by Hayat *et al.* (2008), in a case where the fluid flow was induced due to a pull with constant velocities of the discs.

Various aspect of the flow problems in porous channel have been studied, Bég *et al.* (2009), studied unsteady magnetohydrodynamic Hartmann-Couette flow and heat transfer in a Darcian channel with Hall current, ionslip, viscous and Joule heating effects. Makinde *et al.* (2012) studied unsteady hydromagnetic flow of a reactive variable viscosity third-grade fluid in a channel with convective cooling while Vieru *et al.* (2010) studied the Axial Flow of Several Non-Newtonian Fluids through a Circular Cylinder.

Seth *et al.* (2011), studied the problem considered by Singh and Kumar (1983) when the fluid flow is confined to porous boundaries with suction and injection considering two cases of interest, viz (i) impulsive movement of the lower plate and (ii) uniformly accelerated movement of the lower plate. Seth *et al.* (2011) concluded that the suction exerted a retarding influence on the fluid velocity whereas injection has accelerating influence on the flow while the magnetic field, time and injection reduce shear stress at lower plate in both the cases while suction increases shear stress at the lower plate.

Jha and Apere (2011) investigated Hall and ion-slip effects on unsteady MHD Couette flow in a rotating system with suction and injection. Guchhait *et al.* (2011) studied the combined effects of Hall current and rotation on unsteady Couette flow in porous channel. Sheikholeslami *et al.* (2013) studied Heat transfer of Cu-water nanofluid flow between parallel plates while Prasad *et al.* (2012) considered unsteady hydromagnetic couette flow through a porous medium in a rotating system. Seth *et al.* (2012) studied the effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field and also in the same year Seth *et al* (2012).considered unsteady MHD Couette flow of class-II of a viscous incompressible electrically conducting fluid in a rotating system.

More researchers, Ahmed and Kalita (2013) considered a sinusoidal fluid injection/suction on MHD three dimensional Couette flow through a porous medium in the presence of thermal radiation and also Ahmed and Kalita (2013) studied Magnetohydrodynamic transient flow through a porous medium bounded by a hot vertical plate in presence of radiation.

Extensive researches have been done, including those cited above, on the flow between parallel plates. However, no emphasis has been given to the problems analyzed by Seth *et al.* (2011) with consideration with motion on the upper plate. This work presents findings of studies on MHD Couette flow problem between porous plates with magnetic field lines fixed relative to the moving upper plate with suction and injection on the plates.

1.5 Statement of the Problem

In the previous studies, analysis on MHD Couette flow problem between porous plates with magnetic field lines fixed relative to the moving upper plate with suction and injection on the plates has not been investigated. This study considers the flow of unsteady viscous incompressible electrically conducting fluid between two parallel porous plates $y = 0$ and $y = h$ of infinite length in x and z directions with a constant pressure gradient in the presence of a uniform transverse magnetic field H_o applied parallel to the y axis.

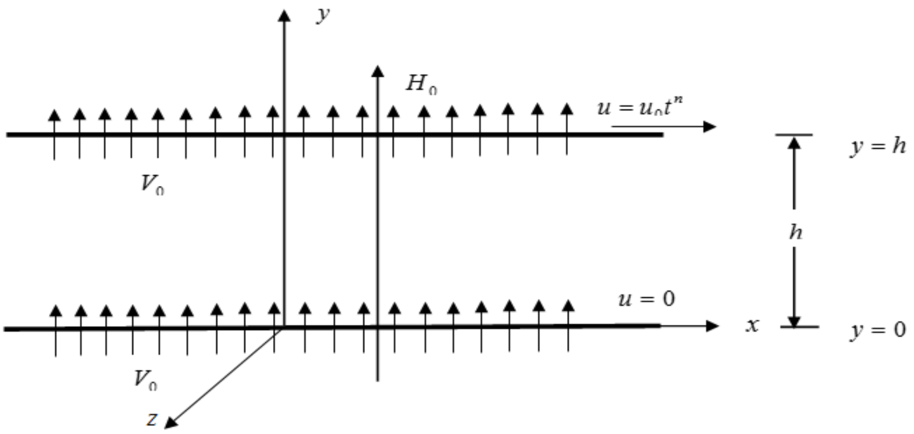

Figure 1.1: Physical Model of the Problem

1.6 Justification of the study

MHD flow has many important varied applications in industries, engineering and other scientific fields. Examples of few applications are; in the modeling of processes such as transpiration cooling, where the walls of a pipe or channel containing heated fluid are protected from overheating by passing cooler fluid over the exterior surface of the pipe or channel; in the modeling of the fluid flow occurring during the separation of isotopes of Uranium-235 and Uranium-238 by gaseous diffusion in order to produce fuel for nuclear reactors; in controlling boundary layer flow over aircraft wings by injection or suction of fluid out of or into the wing, or as part of a model for flow past a membrane or filter

Other important applications of MHD flow between porous plates are in the designing of cooling systems with liquid metals, geothermal reservoirs, in petroleum and mineral industries, underground energy transport, accelerators, MHD generators, pumps, flow meters, purification of crude oil, polymer technology among many other areas. Hydromagnetic Couette flow of an electrically conducting fluid in the presence of a transverse magnetic field is one of the problems in Magnetohydrodynamics which has received considerable attention due to its varied and wide applications in the areas of Geophysics, Astrophysics and fluid engineering fields.

Flow in porous channels finds varied applications in industries, engineering and many other scientific fields.

1.7 Hypothesis

The motion of the upper plate has no effect on the MHD flow between two parallel porous plates with suction and injection and magnetic field lines fixed relative to the moving plate.

1.8 Objectives of the Study

1.8.1 General Objective

To analyze unsteady hydromagnetic couette flow between two parallel porous plates with magnetic field lines fixed relative to the moving upper plate with injection /suction through the walls of the channel.

1.8.2 Specific objectives

1. To determine the effects of magnetic field, pressure gradient and viscosity on the flow variables.
2. To determine the effect of suction /injection on the flow variables.

Having defined the terms that are used in this study and stating the problem, we shall consider the governing equations in the next chapter

CHAPTER TWO
THE GOVERNING EQUATIONS

2.0 Introduction

The equations governing the flow of an incompressible electrically conducting fluid in the presence of a transverse magnetic field lines fixed relative to the moving upper plate with suction and injection on the plates are presented in this chapter. First, this chapter considers the assumptions made in this particular flow problem and the consequences arising due to these assumptions. The equations of conservation of mass and momentum are considered and the equations governing the flow are given in their general forms, non-dimensional parameters are defined and a finite difference scheme to solve the resulting equations is described.

2.1 Assumptions

The following assumptions are made in this study

1. All velocities are small compared with that of light $v^2/c^2 << 1$
2. The fluid flow is restricted to a laminar domain.
3. The fluid is incompressible.
4. Electrical conductivity, thermal conductivity, Dynamic viscosity, Darcy permeability, and diffusion co efficient are constant.
5. The force due to electric field is negligible compared with the force $\mathbf{J} \times \mathbf{B}$ due to magnetic field.
6. The induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible.
7. The porous medium is entropic, homogeneous and non – magnetic, thus there is no magnetic induction.

The fundamentals of fluid dynamics are based on universal laws that govern fluid flows such as the equation of continuity, equation of motion and the electromagnetic equation.

2.2 THE GOVERNING EQUATIONS

2.2.1 Equation of continuity

The equation of continuity is derived from the law of conservation of mass which states that under normal conditions mass can neither be created nor destroyed. It is derived by taking a mass balance on the fluid entering and leaving a volume element in the flow field. The general equation of continuity of a fluid flow is given by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\left(\rho \vec{q}\right) = 0 \qquad (2.1)$$

where $\vec{q}$ is the velocity in x, y and z directions ($\vec{q} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$)

2.2.2 Equation of motion

This equation is also known as the momentum equation and is derived from the Newton's second law of motion. The law requires that the sum of all the forces acting on a control volume must be equal to the rate of change of fluid momentum within the control volume.

$$\frac{\partial \vec{q}}{\partial t} + \vec{q}\left(\vec{\nabla}\vec{q}\right) = -\frac{1}{\rho}\vec{\nabla} P + \upsilon \nabla^2 \vec{q} + \vec{F} \qquad (2.2)$$

where $\frac{\partial \vec{q}}{\partial t}$ is the temporal acceleration , $\vec{q}\left(\vec{\nabla}\vec{q}\right)$is the convective acceleration, $\vec{\nabla} P$ is the pressure gradient , $\upsilon\nabla^2\vec{q}$ is the force due to viscosity and $\vec{F}$ represents the body forces vector in x ,y and z directions.

2.2.3 Electromagnetic equations

The electromagnetic equations give the relationship between **E** the electric field intensity, **B** the magnetic induction vector, **D** the electric displacement, **H** the magnetic field intensity, **J** the induction current density vector and the charge density e_e, as given by Griffiths (1999) are:

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad (2.3)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (2.4)$$

$$\nabla \times \mathbf{E} = \frac{\partial B}{\partial t} \qquad (2.5)$$

Equation (2.3) is Ampere's law, named after the Ampere Andre-Marie, who showed that wires carrying electric currents attract and repel each other magnetically. Equation (2.4) is referred to as the Gauss' law for magnetism which states that all magnetic fields **B** have field lines that are continuous.

Equation (2.5) is known as Faraday's law, named after Michael Faraday, who, in 1831, discovered experimentally that a current is induced in a conducting loop when magnetic flux linking the loop changes. This law states that changing magnetic fields produce an electric field. The electromagnetic field induced in a circuit is equal to the rate of change with time of the total magnetic flux through the circuit no matter how the flux changes.

2.3 Non -Dimensional Numbers

The dimensionless parameters allow the application of the results obtained in a model to any other dynamically similar case. In this work there are two non-dimensional numbers that are used. These are;

- Reynolds number
- Hartmann number

2.3.1 The Reynolds Number, Re

The Reynolds number is the ratio of inertial forces to viscous forces and is important in analyzing any type of flow where there is velocity gradient shear. It is expressed as

$$\mathrm{Re} = \frac{\rho V L}{\vartheta}$$

The Reynolds number indicates the relative significance of the viscous effects compared to the inertia effect. If the Reynolds number of the system is small, the viscous force is predominant and the effect of viscosity is important in the whole flow field otherwise if the Reynolds number is large, the inertia force is predominant and the effect of viscosity is important only in the thin layer of the region near solid boundary.

2.3.2 Hartmann Number, *M*

The Hartmann number is the ratio of the magnetic force to viscous force and is defined as

$$M^2 = \frac{\sigma \mu_e^2 H_0^2 v}{U^2}$$

It is a dimensionless number which gives a measure of the relative importance of drag forces resulting from magnetic induction and viscous forces in Hartmann flow, and determines the velocity profile for such flow.

2.4 Mathematical Formulation

Initially (when time $t \leq 0$), fluid and the porous plates of the channel are assumed to be at rest. When time $t > 0$, the upper plate ($y = h$) starts moving with time dependent velocity $u_o t^n$ (where u_o is a constant and n a positive integer) in the x direction while the lower plate is fixed, the fluid suction/injection takes place through the walls of the channel with uniform velocity V_o where $V_o > 0$ for suction and $V_o < 0$ for injection.

The velocity and the magnetic fields are given as $q = (u, v_0, 0)$ and $\vec{H} \equiv (0, H_0, 0)$ respectively.

The magnetic forces $= \sigma \mu_e^{\ 2} H_0 \times Velocity$

$$J \times B = \begin{vmatrix} i & j & k \\ 0 & 0 & \sigma \mu_e^{\ 2} H_0 u \\ 0 & H_0 & 0 \end{vmatrix} = (-\sigma \mu_e^{\ 2} H_0^{\ 2} u)\hat{i} \qquad (2.6)$$

From the Navier Stokes equation

$$\rho \frac{\partial u}{\partial t} + \rho u \nabla u = -\nabla P + \mu \nabla^2 u + F \qquad (2.7)$$

$$\rho \frac{\partial u}{\partial t} + \rho u \nabla u = -\nabla P + \mu \nabla^2 u + J \times B \qquad (2.8)$$

The flow is incompressible (the density ρ, is considered a constant) and is considered in one dimension along the x- axis hence the Navier stokes equation along the x-axis is given as

$$\rho \frac{\partial u}{\partial t} + \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + J \times B \qquad (2.9)$$

For a Couette flow $-\frac{\partial P}{\partial x} = 0$ but for the analysis $-\frac{\partial P}{\partial x} =$ a constant β^*. The two plates are infinite in length hence $\frac{\partial u}{\partial x} = 0$.The fluid is injected on the lower plate with a constant velocity V_0 and is also sucked from the upper plate at the same constant velocity V_0.The general equation governing the flow reduces to

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\beta^*}{\rho}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{(-\sigma\mu_e{}^2H_0{}^2u)}{\rho} \tag{2.10}$$

where $\beta=\frac{\beta^*}{\rho}$, and

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e{}^2H_0{}^2u}{\rho} \tag{2.11}$$

where $\upsilon=\frac{\mu}{\rho}$

The magnetic field lines are fixed relative the moving upper plate (The upper plate is accelerating uniformly–a function of time) hence the velocity is considered as a relative velocity and reflects how fast the fluid is moving relative to the plate .The general equation governing the flow

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e{}^2H_0{}^2\left(u-u_0t^n\right)}{\rho} \tag{2.12}$$

With the boundary conditions defined as;

$$\begin{aligned} &u=0 && 0\le y\le h && t\le 0\\ &u=u_0t^n && \text{at}\quad y=h && t>0\\ &u=0 && \text{at}\quad y=0 && t>0 \end{aligned} \tag{2.13}$$

Taking $n=1$, for a case of uniform acceleration, the governing equation for the flow becomes

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e{}^2H_0{}^2\left(u-u_0t\right)}{\rho} \tag{2.14}$$

2.5 Non-Dimensionalization of the Equations

Non dimensionalization is a process that aims at ensuring that the results obtained from a study are applicable to other geometrically similar configurations under similar set of conditions. The method is of great generality and mathematical simplicity and starts with selecting a suitable scale against which all dimensions in a given physical model are scaled. The non dimensionalization of the governing equation is performed by selecting characteristic dimensionless quantities. The independent variables are non-dimensionalized according to the following dimensionless quantities.

$$y^* = \frac{y}{h} = \frac{L}{L} \qquad \text{(Dimensionless)} \qquad (2.15)$$

$$u^* = \frac{uh}{\upsilon} = \frac{LT^{-1}L}{L^2T^{-1}} \qquad \text{(Dimensionless)} \qquad (2.16)$$

$$t^* = \frac{t\upsilon}{h^2} = \frac{TL^2T^{-1}}{L^2} \qquad \text{(Dimensionless)} \qquad (2.17)$$

The dimensionless quantities used in non dimensionalization of the governing equation (2.14) and the boundary condition (2.13) are

$$y^* = \frac{y}{h}, \qquad u^* = \frac{uh}{\upsilon} \qquad \text{and} \qquad t^* = \frac{t\upsilon}{h^2} \qquad (2.18)$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial y^*}\frac{\partial y^*}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{\upsilon}{h}\frac{\partial u^*}{\partial t^*}\frac{\upsilon}{h^2} = \frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*} \qquad (2.19)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y} = \frac{\upsilon}{h}\frac{\partial u^*}{\partial y^*}\frac{1}{h} = \frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*} \qquad (2.20)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}\right) = \frac{\partial}{\partial y^*}\left(\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}} \qquad (2.21)$$

Replacing on the governing equation (2.14)

$$\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*} + V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*} = \beta + \upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\mu_e{}^2 H_0{}^2\left(u - u_0 t\right)}{\rho} \qquad (2.22)$$

Non dimensionalizing the relative velocity in equation (2.22) by setting $u^* = \frac{u}{\upsilon}h \Rightarrow u = \frac{\upsilon u^*}{h}$ and $t^* = \frac{t\upsilon}{h^2} \Rightarrow t = \frac{t^* h^2}{\upsilon}$

Substituting in (2.22) to non-dimensionalize the the relative velocity

$$\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\beta+\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e{}^2H_0{}^2}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right)\text{(2.23)}$$

and multiplying the equation by $\frac{h^3}{\upsilon^2}$ gives

$$\frac{h^3}{\upsilon^2}.\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+\frac{h^3}{\upsilon^2}.V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{h^3}{\upsilon^2}.\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e{}^2H_0{}^2}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right)\text{ (2.24)}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e{}^2H_0{}^2}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right)\text{(2.25)}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e{}^2H_0{}^2}{\rho}.\frac{1}{h}\left(\upsilon u^*-u_0\frac{t^*h^3}{\upsilon}\right)\text{(2.26)}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e{}^2H_0{}^2h^2}{\rho\upsilon^2}\left(\upsilon u^*-u_0\frac{t^*h^3}{\upsilon}\right)\qquad\text{(2.27)}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e{}^2H_0{}^2h^2}{\rho\upsilon^2}.\upsilon\left(u^*-u_0\frac{t^*h^3}{\upsilon^2}\right)\text{ (2.28)}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e{}^2H_0{}^2h^2}{\rho\upsilon}\left(u^*-u_0\frac{t^*h^3}{\upsilon^2}\right)\text{(2.29)}$$

The expression $\frac{\sigma\mu_e{}^2H_0{}^2h^2}{\rho\upsilon}=M^2$ is the Hartmann number squared, and $\frac{u_0h}{\upsilon}$ is the Reynolds number Re and hence substituting in Equation 2.29, this gives

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-u_0\frac{t^*h^3}{\upsilon^2}\right)\qquad\text{(2.30)}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-\frac{\mathrm{Re}\,h}{\upsilon}t^*\right)\qquad\text{(2.31)}$$

Equation (2.31) is the governing equation in non-dimensional form.

The non-dimensional boundary conditions from (2.13) using the non-dimensional parameters from equations (2.15), (2.16) and (2.17) are obtained as

$$u^* = 0 \qquad 0 \le y \le 1 \text{ and } t^* \le 0$$

$$u^* = \frac{t^* h}{\upsilon} \mathrm{Re} \quad \text{at } y^* = 1; \qquad t^* > 0 \qquad (2.32)$$

$$u^* = 0 \quad \text{at} \quad y^* = 0\ ; \qquad t^* > 0$$

The governing equation in non-dimensional form (2.31) together with the boundary conditions (2.32) are presented in their finite difference forms consistent with the method of solution in the next chapter.

CHAPTER THREE

METHOD OF SOLUTION AND ANALYSIS

3.0 Introduction

In this chapter, the method of solution is discussed and the governing equations are presented in their finite difference forms consistent with the method of solution. The final set of the equations are presented in this chapter in their finite difference form.

3.1 Finite Difference Technique

The finite difference approximations for derivatives are one of the methods that can be used to solve differential equations. The principle of finite difference methods is close to the numerical schemes used to solve ordinary and partial differential equations and consists of approximating the differential operator by replacing the derivatives in the equation using difference quotients. The domain is partitioned in space and time and approximations of the solution are computed at the space or time points.

The governing equations together with the boundary conditions are solved numerically because of the nonlinear nature of the equations that are obtained. The finite difference scheme is consistent since the operator reduces to the original differential equations being solved as the increments in the independent variables vanish and based on reasonable approximations of the derivatives.

The solution by the finite-difference method approaches the true solution to the partial differential equation as the increments on the mesh are minimized and approach zero hence a faster convergence rate. In this method, the (t, y) plane is divided into a network of rectangles of sides $\Delta t = h$ and $\Delta y = k$ by drawing the set of lines:

$t = ih$ and $y = jk$ where i , $j = 0,1,2,.........$

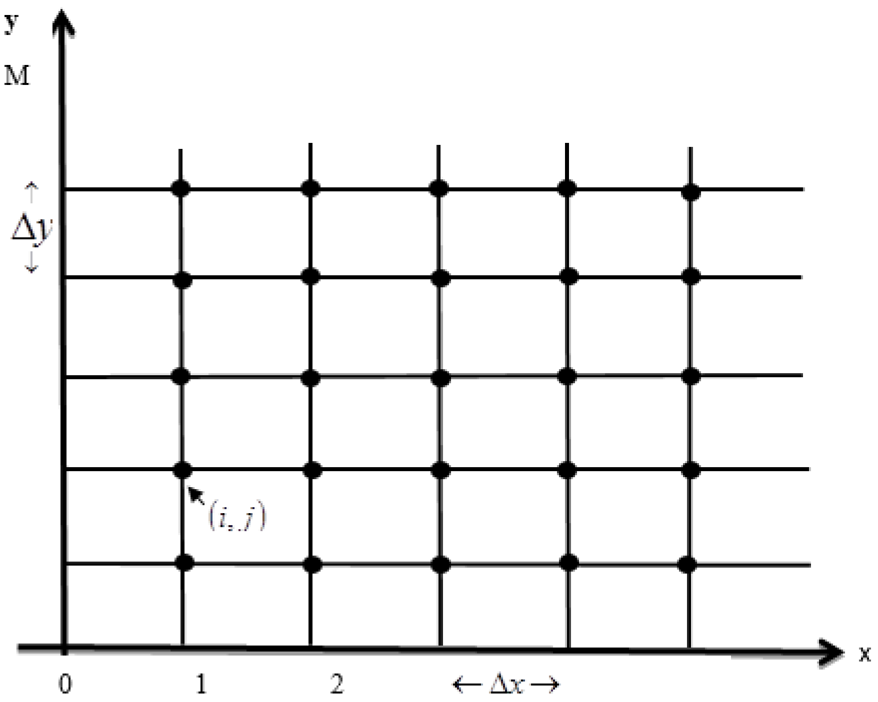

Figure 3.1: Finite difference mesh.

The finite difference mesh is used to divide the physical flow domain into finite number of discrete approximation for space and time domains to be used in the finite difference method.

Consider the plane in Figure 2. Each corner of the cell forms the mesh or grid point .Consider a reference point (i, j) where i and j represent t and y respectively. Using the notation $(i \pm 1)$ for $(t \pm \Delta t)$and $(j \pm 1)$ for $(x \pm \Delta x)$ we define the adjacent points that are i and j units from the reference point and give their co-ordinates in terms of i and Δt along the x-axis, j and Δx along the y-axis. In finite difference approximation the derivatives are replaced with the finite differences. $U = u(t, x)$ and the points of intersection of these families of lines are called mesh points or grid points. Then, by central differencing, the first and second order derivatives with respect to t are obtained in finite difference form. A finite difference mesh is used to express the unknown functional values at the $(i, j)^{th}$ interior mesh using the known boundary points.

The finite difference analogues of the PDEs arising from the equation governing this flow are obtained by replacing the derivatives in the governing equations by their corresponding difference approximation taking into account the initial values and boundary values set.

$$\frac{\partial u^*}{\partial t^*} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta t} \quad (3.1)$$

$$\frac{\partial u^*}{\partial y^*} = \frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y} \quad (3.2)$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{u_{i,j+1} + u_{i,j-1} - 2u_{i,j}}{(\Delta y)^2} \quad (3.3)$$

3.2 Governing Equation in Finite Difference Form

Crank Nicolson proposed a method in 1957 in which the second derivative is replaced by the average of the finite difference approximation on the j^{th} and the $(j+1)^{th}$ row thus for the governing equation $\frac{\partial u^*}{\partial t^*} + \frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*} = \frac{h^3}{\upsilon^2}\beta + \frac{\partial^2 u^*}{\partial y^{*2}} - M^2\left(u^* - \frac{\mathrm{Re}\, h}{\upsilon}t^*\right)$ (3.4)

The following substitutions are done for the derivatives for the Crank Nicolson, we have the proposed averages as

$$u^* = \frac{u_{i,j+1} + u_{i,j}}{2} \quad (3.5)$$

$$\frac{\partial u^*}{\partial t^*} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \quad (3.6)$$

$$\frac{\partial u^*}{\partial y^*} = \frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4(\Delta y)} \quad (3.7)$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} \quad (3.8)$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{1}{2}\left(\left\{\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2}\right\} + \left\{\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta y)^2}\right\}\right) \quad (3.9)$$

Replacing in the governing equation

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}+\frac{V_0 h}{\upsilon}\left(\frac{u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}}{4(\Delta y)}\right)=\frac{h^3}{\upsilon^2}\beta+$$
$$\left[\frac{1}{2}\left(\left\{\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2}\right\}+\left\{\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}}{(\Delta y)^2}\right\}\right)\right] \quad (3.10)$$
$$-M^2\left(\frac{u_{i,j+1}+u_{i,j}}{2}-\frac{\mathrm{Re}\,h}{\upsilon}t_j\right)$$

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}+\frac{V_0 h}{\upsilon}\left(\frac{u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}}{4(\Delta y)}\right)=\frac{h^3}{\upsilon^2}\beta+$$
$$\frac{1}{2(\Delta y)^2}\left[\left(u_{i-1,j}-2u_{i,j}+u_{i+1,j}\right)+\left(u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}\right)\right] (3.11)$$
$$-M^2\left(\frac{u_{i,j+1}+u_{i,j}}{2}-\frac{\mathrm{Re}\,h}{\upsilon}t_j\right)$$

Multiplying through by Δt

$$\left(u_{i,j+1}-u_{i,j}\right)+\frac{V_0 h\Delta t}{\upsilon}\left(\frac{u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}}{4(\Delta y)}\right)=\frac{h^3\Delta t}{\upsilon^2}\beta+$$
$$\frac{1\Delta t}{2(\Delta y)^2}\left[\left(u_{i-1,j}-2u_{i,j}+u_{i+1,j}\right)+\left(u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}\right)\right] \quad (3.12)$$
$$-M^2\Delta t\left(\frac{u_{i,j+1}+u_{i,j}}{2}-\frac{\mathrm{Re}\,h}{\upsilon}t_j\right)$$

$$\left(u_{i,j+1}-u_{i,j}\right)+\frac{V_0 h\Delta t}{4(\Delta y)\upsilon}\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=\frac{h^3\Delta t}{\upsilon^2}\beta+$$
$$\frac{1\Delta t}{2(\Delta y)^2}\left[u_{i-1,j}-2u_{i,j}+u_{i+1,j}+u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}\right]-\frac{M^2\Delta t}{2}\left(u_{i,j+1}+u_{i,j}\right)+$$
$$M^2\Delta t\frac{\mathrm{Re}\,h}{\upsilon}t_j$$

(3.13)

Rearranging (3.13) gives

$$\left(u_{i,j+1}-u_{i,j}\right)+\frac{V_0 h\Delta t}{4\upsilon(\Delta y)}\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=\frac{h^3\Delta t}{\upsilon^2}\beta+$$
$$\frac{1\Delta t}{2\left(\Delta y\right)^2}\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)-\frac{M^2\Delta t}{2}\left(u_{i,j+1}+u_{i,j}\right)+$$
$$M^2\Delta t\frac{\operatorname{Re}h}{\upsilon}t_j$$

(3.14)

Here $A=-\frac{V_0 h\Delta t}{4\upsilon(\Delta y)}$, $B=\frac{h^3\Delta t}{\upsilon^2}\beta$, $C=\frac{1\Delta t}{2\left(\Delta y\right)^2}$, $D=\frac{M^2\Delta t}{2}$, $E=M^2\Delta t\frac{\operatorname{Re}h}{\upsilon}$ and the the suction/ injection parameter $S=\frac{V_0 h}{\upsilon}$.

Substituting the values of A, B, C, D, E and S in (3.14) gives

$$\left(u_{i,j+1}-u_{i,j}\right)-A\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=B+$$
$$C\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)-D\left(u_{i,j+1}+u_{i,j}\right)+$$
$$Et_j$$

(3.15)

Rearranging (3.15) gives

$$u_{i,j+1}-u_{i,j}-Au_{i+1,j+1}+Au_{i-1,j+1}-Au_{i+1,j}+Au_{i-1,j}=B+$$
$$Cu_{i+1,j+1}-2Cu_{i,j+1}+Cu_{i-1,j+1}+Cu_{i+1,j}-2Cu_{i,j}+Cu_{i-1,j}$$
$$-Du_{i,j+1}-Du_{i,j}+Et_j$$

(3.16)

Rearranging equation (3.16) gives

$$u_{i,j+1}-Au_{i+1,j+1}+Au_{i-1,j+1}-Cu_{i+1,j+1}+Cu_{i-1,j+1}+2Cu_{i,j+1}+Du_{i,j+1}=B+$$
$$u_{i,j}-Au_{i-1,j}+Au_{i+1,j}+Cu_{i+1,j}-2Cu_{i,j}+Cu_{i-1,j}-Du_{i,j}+Et_j$$

(3.17)

Collecting the like terms form equation (3.16) gives

$$(1+2C+D)u_{i,j+1}-(A+C)u_{i+1,j+1}+(A+C)u_{i-1,j+1}$$
$$=B+(1-2C+D)u_{i,j}+Cu_{i+1,j}+(C-A)u_{i-1,j}+Et_j \qquad (3.18)$$

Rearranging equation (3.18)

$$-(A+C)u_{i+1,j+1}+(1+2C+D)u_{i,j+1}+(A+C)u_{i-1,j+1}=$$
$$Cu_{i+1,j}+(1-2C+D)u_{i,j}+(C-A)u_{i-1,j}+Et_j+B \qquad (3.19)$$

The finite difference equations obtained at any space node, say, i at the time level t_{j+1} has only three unknown coefficients involving space nodes at $i-1, i$ and $i+i$ at t_{j+1}. In matrix notation, these equations can be expressed as $AU = B$ where U is the unknown vector of order $(N-1)$ at any time level t_{j+1}. B is the known vector of order $(N-1)$ which has the value of U at the n^{th} time level and A is the coefficient square matrix of order $(N-1)\times(N-1)$ which is a tridiagonal structure.

The coefficients of the interior nodes will be represented as:

$$\begin{array}{lll} a_j=-(A+C) & d_j=(C-A)u_{i-1,j} & g_j=Et_j \\ b_j=(1+2C+D) & e_j=(1-2C-D)u_{i,j} & h=B \\ c_j=(A+C) & f_j=Cu_{i+1,j} & \end{array} \qquad (3.20)$$

For $j = 2,3,4....(N-1)$, then the equation (6.6) becomes

$$a_j u_{i+1,j+1}+b_j u_{i,j+1}+c_j u_{i-1,j+1}=d_j+e_j+f_j+g_j+h \qquad (3.21)$$

The system of equations resulting from equation (6.8) are represented in a tridiagonal matrix form as

$$\begin{bmatrix} a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 \\ 0 & a_3 & b_3 & c_3 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & a_{N-1} & b_{N-1} & c_{N-1} \end{bmatrix} \begin{bmatrix} u_{1,j+1} \\ u_{2,j+1} \\ \vdots \\ u_{3,j+1} \end{bmatrix} = \begin{bmatrix} d_2 \\ d_3 \\ \vdots \\ d_{N-1} \end{bmatrix} + \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix} + \begin{bmatrix} f_2 \\ f_2 \\ \vdots \\ f_{N-1} \end{bmatrix} + \begin{bmatrix} g_2 \\ g_3 \\ \vdots \\ g_{N-1} \end{bmatrix} + \begin{bmatrix} h \\ h \\ \vdots \\ h \end{bmatrix}$$

(3.22)

Equation (3.23) is the final set of equation and is solved using a computer code in MATLAB software. The results of the simulation in the computer code and discussions are presented in the next chapter.

CHAPTER FOUR

RESEARCH RESULTS AND DISCUSSIONS

4.0 Introduction

In this chapter, the results of the simulations are presented followed by discussions at each step. The simulations are curried out using ISO FLUIDS 3448 which are industrial oils whose kinematic viscosities range between 2 and 10.

4.1 Results and Discussions

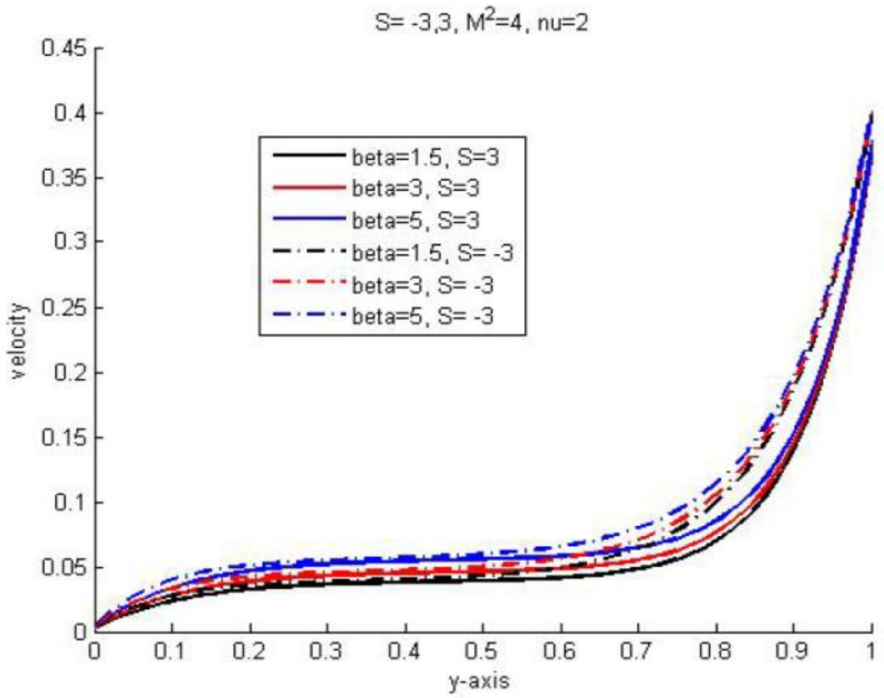

Figure 4.1: Varying the pressure gradient.

From figure 4.1 it is observed that the velocity profiles increases with increase in the pressure gradient (beta) for both the cases of injection and suction. The pressure gradient is applied in the direction of the flow hence an increase in pressure gradient results in an increase in the force in the fluid in the direction of the flow which results in increased velocity of the fluid. The velocities for the injection case , $(S > 0)$ are greater than for the suction case $(S < 0)$ .Injection increases the pressure which increases the force in the fluid hence an increase in the velocities .Injection increases the pressure which increase the force in the fluid hence an increase in the velocities while suction reduces the pressure which reduces the force in the fluid hence a decrease in the velocities which explains why the injection velocities are greater than the suction velocities.

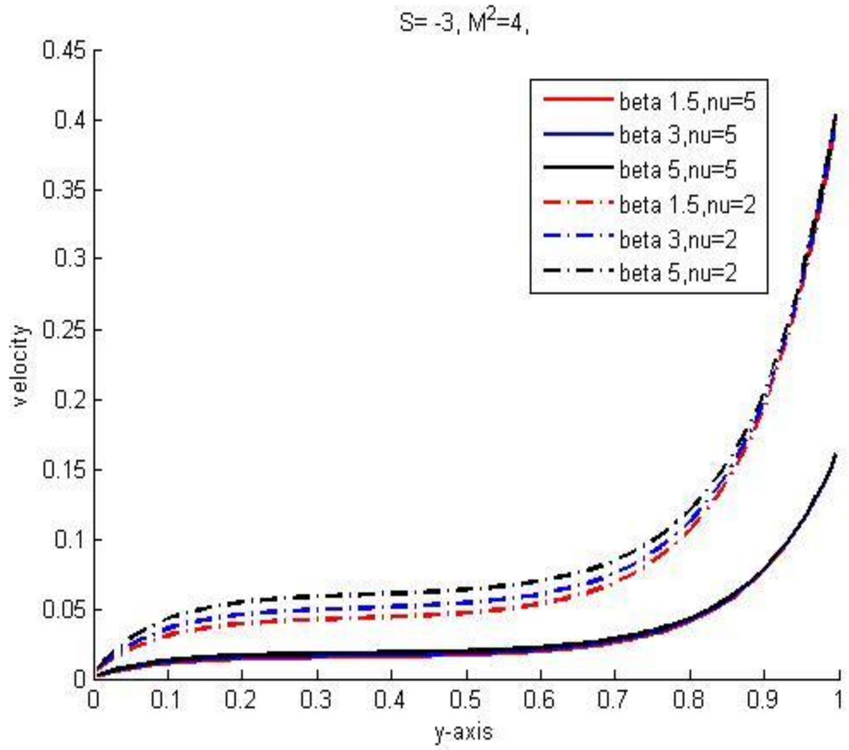

Figure 4.2: Varying the kinematic viscosity and pressure gradient.

Figure 4.2 depicts that the velocity profiles increasing with increase in the pressure gradient for varying kinematic viscosity. There is rapid increase in velocity of the fluid with increase in pressure gradient for small kinematic viscosity as compared to large kinematic viscosity. The effect of pressure gradient decreases with increase in kinematic viscosity. The increase in the kinematic viscosity leads to increase in the frictional forces which oppose the fluid motion.

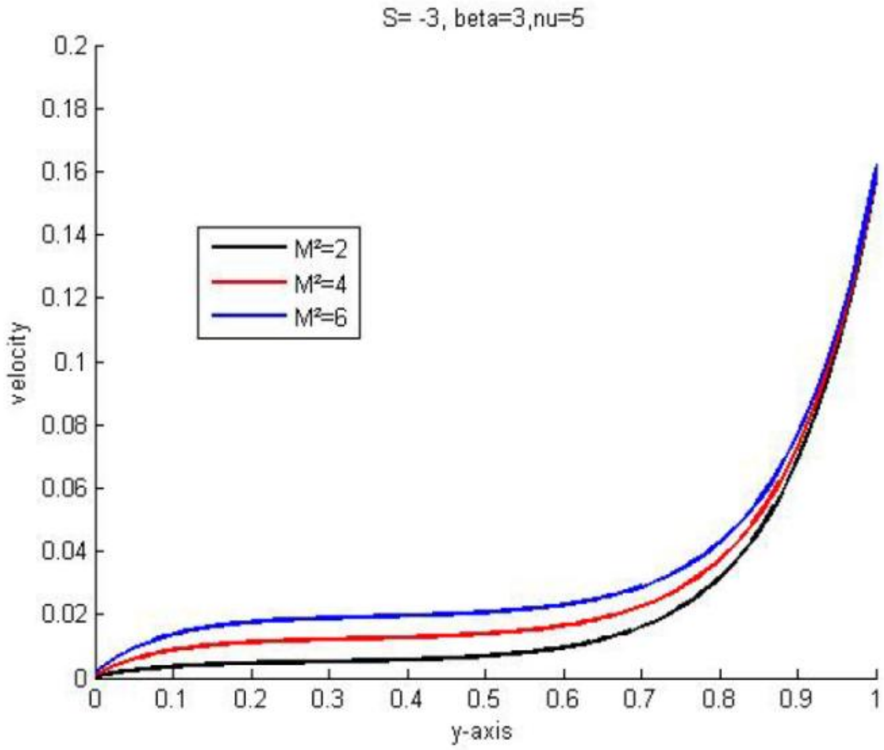

Figure 4.3: Varying the Hartman number, M^2

From figure 4.3 The velocity increases with the increase in the Hartman number .The Hartmann number gives a measure of the relative importance of drag forces resulting from magnetic induction and viscous forces hence an increase in the Hartmann number reduces the drug forces hence increased velocities.

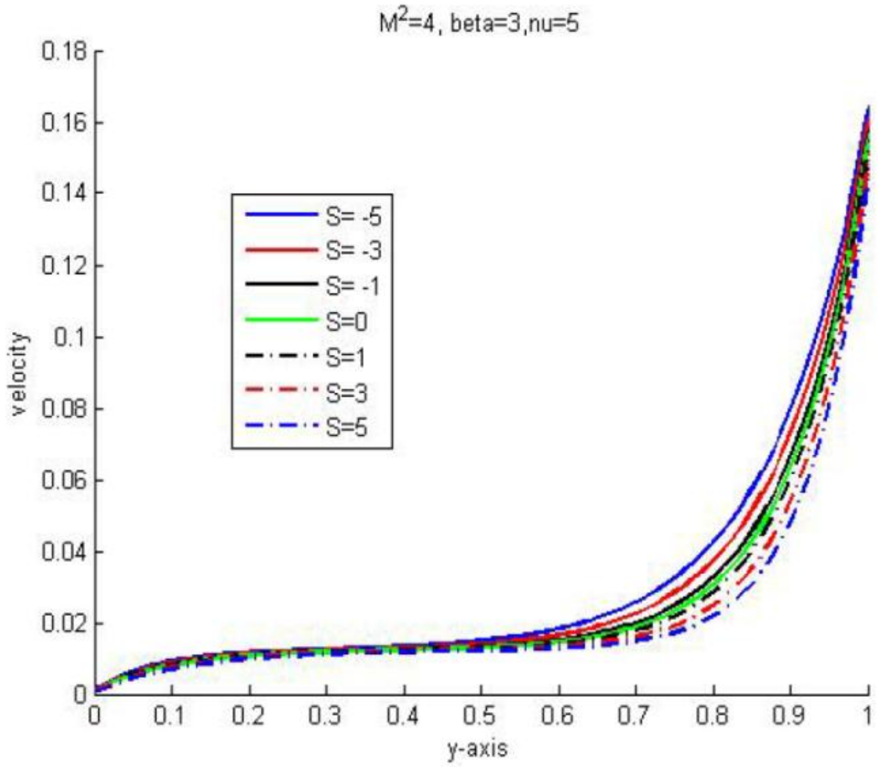

Figure 4.4: Varying the Injection/Suction, S.

From Figure 4.4, an increase in the suction parameter $(S > 0)$ leads to a decrease in the velocity of the fluid. An increase in the injection parameter $(S < 0)$ leads to an increase in the velocity of the fluid. An increase in the suction parameter $(S > 0)$ reduces the pressure which reduces the force hence decrease in the velocity with increase in the suction parameter while an increase in the injection parameter $(S < 0)$ increases the pressure which increases the force in the fluid hence increased velocities. Thus suction exerts a retarding influence on the fluid velocity whereas injection has an accelerating influence on it.

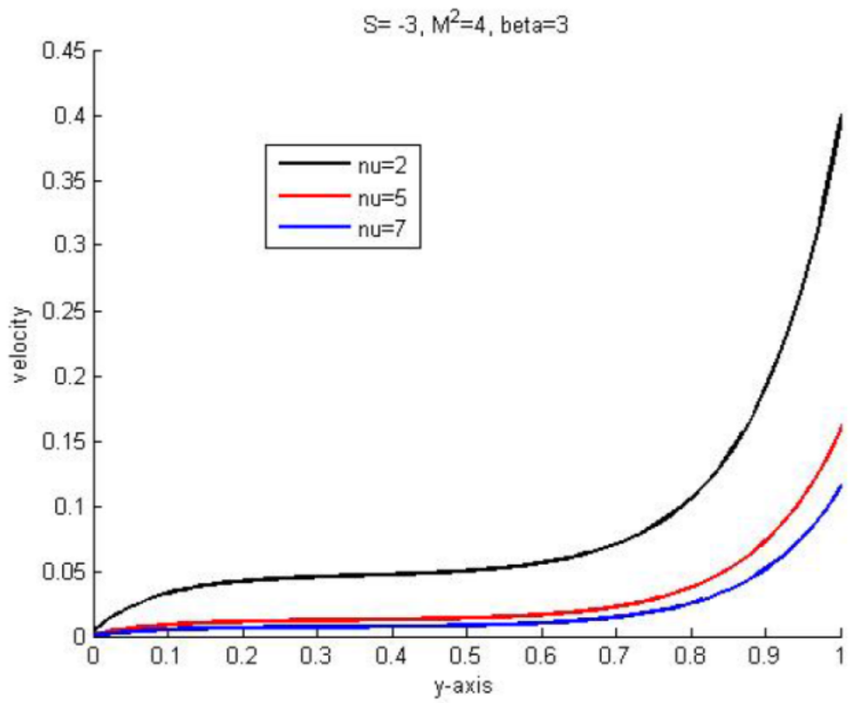

Figure 4.5: Varying the kinematic viscosity.

From Figure 4.5. The velocity of the fluid decreases with the increase in the viscosity of the fluid. Increase in the viscosity of the fluid leads to increase in the viscous forces in the fluid hence decrease in the velocity of the fluid.

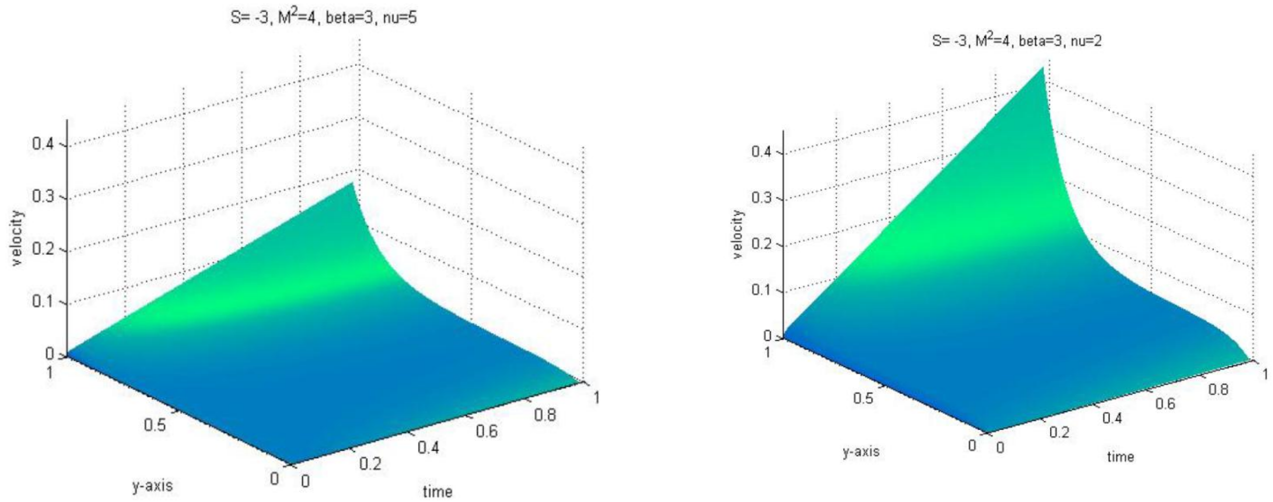

Figure 4.6: Surface plots.

Figure 4.6 are the three dimension shots of the results.

CHAPTER FIVE

VALIDATION CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

This chapter presents the validation of the results of this research study, the conclusion and the recommendations on areas that require further research.

5.1 Validation of the Results

According to the results discussed, we note that the magnetic field, pressure gradient, time and injection have an accelerating influence whereas suction and viscosity exerts a retarding influence on the fluid flow between parallel porous plates with injection/suction with a constant pressure gradient applied in the direction of the flow.

In the absence of the pressure gradient, the results of this study are in agreement with the results of the study by Seth *et al* (2011) which noted that suction and viscosity exert a retarding influence on the fluid velocity whereas injection, magnetic field and time have an accelerating influence.

5.2 Conclusions

The results of this study leads to conclusion that;

(i) Magnetic field, pressure gradient, time and injection have an accelerating influence on the fluid flow with a constant pressure gradient in the direction of the flow on both cases of suction and injection while viscosity and suction exert a retarding influence.

(ii) Fluid velocity in both the cases of suction and injection decreases with increase in the suction parameter and increases with the increase in the injection parameter.

(iii) Suction exerts retarding influence on the fluid velocity whereas injection has an accelerating influence on the fluid velocity.

(iv) Viscosity exerts a retarding influence on the fluid velocity.

5.2 Recommendations

In this study, magnetohydrodynamic flow between two parallel porous plates with injection and suction in the presence of a uniform transverse magnetic field with the magnetic field lines fixed relative to the moving plate with constant pressure gradient has been investigated. It is recommended that further research should be carried out on the flow when variable pressure gradient in the direction of the flow is used and also when variable magnetic field is used between the two parallel porous plates with injection and suction.

Further research should be carried out on MHD flow between two parallel porous plates with injection and suction when the pressure gradient in applied in the direction against the flow with varying magnetic field lines.

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APPENDICES

APPENDIX 1: COMPUTER CODE IN MATLAB

The governing equation (6.9) in matrix notation in finite difference form was simulated in the following computer program code developed using MATLAB software, subject to the boundary conditions as discussed herein. The results were obtained by varying various flow parameters, notably kinematic viscosity, magnetic number, pressure gradient and the suction/injection parameter.

```
function RichardCN()

clear all; clc;
v0=1;h=1;beta=5;M=sqrt(4);niu=2;Re=1;S=-3;
ylow=0;yup=h+.01;ny=151;
dy=(yup-ylow)/(ny-1);
y=ylow:dy:yup;
tinit=0;tend=1;nt=51;
dt=(tend-tinit)/(nt-1);
t=tinit:dt:tend;
u=zeros(ny,nt);rhs=zeros(ny,nt);
%==initial conditions
u(:,1)=0;
%==initial conditions
%==boundary conditions
for j=ny-1:ny
u(j,1:nt)=(h/niu)*Re*t(1:nt);% for plate at y=1
end
u(1:2,1:nt)=0;% for plate at y=0
%==boundary conditions
a=(S/(4*dy))-1/(2*(dy^2));b=(1/dt)+(M*M)/2+(1/(dy^2));c=(-S/(4*dy))-1/(2*(dy^2));
A=diag(diag(b*ones(ny)),0)+diag(a*ones(ny-1,1),1)+diag(c*ones(ny-1,1),-1);
for i=2:nt-1
   for j=2:ny-1
B(j,i)=(((h^3)*beta)/(niu^2))*ones(size(u(j,i)));
%B(ny,i)=(((h^3)*beta)/(niu^2))*ones(size(u(ny,i)));
```

```
rhs(j,i)=Un(u(j+1,i),u(j,i),u(j-1,i),a,b,c,B(j,i),M,Re,h,niu,dt,t(i));
%rhs(ny,i)=Un(0,u(ny,i),u(ny-1,i),a,b,c,B(ny,i),M,Re,h,niu,dt,t(i));
    end
A2(:,:,i)=A;rhs(:,i);
%[L(:,:,i),U(:,:,i)]=lu(A2(:,:,i));
%u(:,i+1)=U(:,:,i)\(L(:,:,i)\rhs(:,i));
[Q(:,:,i),R(:,:,i)]=qr(A2(:,:,i));
A2(:,:,i)=Q(:,:,i)*R(:,:,i);
u(:,i)=R(:,:,i)\(Q(:,:,i)\rhs(:,i));
end
u;
figure(1)
%%subplot(2,1,1)
surfl(t(2:nt-2),y(2:ny-2),u(2:ny-2,2:nt-2))
shading interp
colormap(winter)
xlabel('time')
ylabel('y-axis')
zlabel('velocity')
title('S= -3, M^2=2, beta=3, nu=2')
figure(2)
%%subplot(2,2,3)
hold on
plot(y(2:ny-2),u(2:ny-2,ceil(0.95*(nt-2))),'k','linewidth',2)
%axis([0 1 0 1])
title('S= -3, M^2=4,')
xlabel('y-axis');ylabel('velocity')
hold off
% subplot(2,2,4)
% hold on
% plot(t(2:nt-2),u(ceil(0.95*(ny-2)),2:nt-2),'r','linewidth',2)
% %axis([0 1 0 1])
% xlabel('time');ylabel('velocity')
% hold off
```

```
function rhs=Un(uR,uC,uL,A,B,C,Bb,M,Re,H,Niu,dt,tt)
  rhs=-A*uR+((2/dt)-B)*uC-C*uL+Bb+M*M*Re*((H*H)/Niu)*tt;
end
end
%%
% function RichardCN()
% clear all; clc;
% v0=1;h=1;beta=5;M=sqrt(2);niu=0.03;Re=1;S=-5;
% ylow=0;yup=h;ny=151;
% dy=(yup-ylow)/(ny-1);
% y=ylow:dy:yup;
% tinit=0;tend=1;nt=51;
% dt=(tend-tinit)/(nt-1);
% t=tinit:dt:tend;
% u=zeros(ny,nt);rhs=zeros(ny,nt);
% %==initial conditions
% u(:,1)=0;
% %==initial conditions
% %==boundary conditions
% for j=ny-1:ny
% u(j,1:nt)=(h/niu)*Re*t(1:nt);% for plate at y=1
% end
% u(1:2,1:nt)=0;% for plate at y=0
% %==boundary conditions
% a=(S/(4*dy))-1/(2*(dy^2));b=(1/dt)+(M*M)/2+(1/(dy^2));c=(-S/(4*dy))-1/(2*(dy^2));
% for i=2:nt-1
%    for j=2:ny-1
% B(j,i)=(((h^3)*beta)/(niu^2))*ones(size(u(j,i)));
% B(ny,i)=(((h^3)*beta)/(niu^2))*ones(size(u(ny,i)));
% rhs(j,i)=Un(u(j+1,i),u(j,i),u(j-1,i),a,b,c,B(j,i),M,Re,h,niu,dt,t(i));
% rhs(ny,i)=Un(0,u(ny,i),u(ny-1,i),a,b,c,B(ny,i),M,Re,h,niu,dt,t(i));
%    end
% end
% b2=b*ones(ny,nt);a2=a*ones(ny,nt);c2=c*ones(ny,nt);
```

```
%
% for i=2:nt
%     for j=2:ny
%     c2(j,i)=c2(j,i)/b2(j-1,i);
%     b2(j,i)=b2(j,i)-c2(j,i)*a2(j-1,i);
%     rhs(j,i)=rhs(j,i)-c2(j,i)*rhs(j-1,i);
%     end
% u(ny,i)=rhs(ny,i)/b2(ny,i);
% for j=ny-1:-1:1
%  u(j,i)=(rhs(j,i)-a2(j,i)*u(j+1,i))/b2(j,i);
% end
% end
% u;
% figure(1)
% subplot(2,1,1)
% mesh(t(2:nt-2),y(2:ny-2),u(2:ny-2,2:nt-2))
% xlabel('time')
% ylabel('y-axis')
% zlabel('velocity')
% %figure(2)
% subplot(2,2,3)
% hold on
% plot(y(2:ny-2),u(2:ny-2,ceil(0.95*(nt-2))),'r','linewidth',2)
% xlabel('y-axis');ylabel('velocity')
% hold off
% subplot(2,2,4)
% hold on
% plot(t(2:nt-2),u(ceil(0.95*(ny-2)),2:nt-2),'r','linewidth',2)
% xlabel('time');ylabel('velocity')
% hold off
% function rhs=Un(uR,uC,uL,A,B,C,Bb,M,Re,H,Niu,dt,tt)
%    rhs=-A*uR+((2/dt)-B)*uC-C*uL+Bb+M*M*Re*((H*H)/Niu)*tt;
% end
% end
```