A Review of Ball mill grinding process modeling using Discrete Element Method

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Abstract—In the two past decades, the discrete element method (DEM) has been used to model the grinding mill and grinding media motion in mineral processing industries. There have been many different discrete element method models that provided good prediction of granular material behavior mainly the trajectory of particles, contact force, kinetic energy, and power draw of ball mill. This paper is intended to review the ball milling parameters that affect the grinding performance of a ball mill mainly the power consumption and the throughput quality. Particularly, the discrete element method (DEM) and its use to optimize critical parameters, limitations and achievement and identifying areas that still need further research.

Keywords—Ball mill, comminution, discrete element method, tumbling mill

I. INTRODUCTION

I N mineral processing, valuable ore minerals need to be liberated from the gangue in order to achieve a product with desirable grade after concentration processes. The release of these valuable minerals is obtained through comminution which is done in a grinding mill or a crusher machine. The process of size reduction also known as comminution is highly energy intensive and expensive. It is estimated that industrial comminution processes absorb from 3 to 5% of global electric energy consumption [1]. Fig. 1 shows the typical cost of



Fig. 1. Contribution of current energy use by equipment across the mining industry. [2]

grinding mill influenced by energy consumed, type of liner used, and the grinding media. The total milling cost (energy,

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grinding media, liner/lifters, and labor cost) is affected by the mill liner/lifter particularly in autogenous (AG) and semiautogenous(SAG) the energy usage in mining industries [2]. The total grinding cost was found to be 40 of the total cost in mining industries. Ball mill grinds material by rotating a cylinder with grinding balls, causing the balls to fall back into the cylinder and onto the material to be ground as shown in Fig. 2 [3]. Collision impact reduce particle size as a result of kinetic energy and potential energy of balls. Grinding mills are able to reduce size particles on a relatively wide range of particle sizes, hence, their wide applicability in the industry, production of noncrystalline materials and in research laboratories [4]. The movement of grinding media and granular material is affected by different parameters: the design of the mill drum (the mill diameter, the length, the size of the liner, and lifters); grinding media (ball size and their size distribution, ball material); Mill filling (the charge volume, feed size) critical speed and grinding time will affect the efficiency of the ball mill [5]. Researches aimed at understanding the grinding parameters that affect the grinding mechanisms have been conducted, estimating power draft, and modeling milling process. Discrete element method (DEM) has made a great contribution in modeling and understanding the grinding problem. For instance, DEM can be used to model the collisions of individual materials in the drum, which when applied to the entire charge mass over a period of time results in the mass charge motion. DEM is also quite reliable because the underlying principles originate from the fundamental laws of physics, provides an insight into the charge motion, and simultaneously gives other information, such as distribution of impact energy, force transmission, and stresses on the wall, etc [6]. This paper seeks to review different methods used to model the grinding process of a ball mill as well as to evaluate the effect of lifter geometry, grinding media size and mill speed on power consumption of grinding mill.

II. THE DISCRETE ELEMENT METHOD

The discrete element method is a numerical modeling technic that allows to describe the mechanical behavior of distinct materials which interact with their nearest neighbor through local contact laws. DEM uses two simple theories: Newtons Second Law and a force displacement law. The force displacement law calculates forces at the contacts between particles, and then the effect of these forces on the each particle is determined from Newtons Second Law [7]. The update of the position of each particle are then used to calculate the new contact forces and this cycle is repeated for each time step.

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Fig. 2. Illustration of ball mill grinding mechanis [3]

Thus these two contact laws are used to trace the movement of the particles This process is summarized in Fig. 3.



Fig. 3. DEM Calculation Cycle

Cundall and Strack [7] were the first to use the discrete element method, the method was based on the use of an explicit numerical scheme in which the interaction of the particles materials can be monitored contact by contact and the motion of each particles can be modeled. Since then DEM has been adapted to suit many other applications such as, granular flow, powder mixing, and in modeling of many physical systems. Lorig et al [8], analyzed the rock-support interaction, Campbell et al [9], used DEM for granular shear flow analysis, John et al [10], used DEM based on two shaped element to model granular soil behavior, Mishra et al [11], simulated the behavior of balls in a milling machine using DEM, Raasch et al [12], analyzed the trajectories and impact velocities of grinding bodies in planetary ball mill, Datta et al [13] analyzed the power draw in ball mills using the discrete element method, Kim et al [14] analyzed ball movement for research of grinding mechanism of a stirred ball mill with 3D discrete element method.

In DEM particles are usually modeled either in two dimensional (circular discs) or three dimensional (spherical). However, the particles shape can also be modeled in an other shape such as ellipsoid [15], and polygon [16], as well as irregular shape which can be modeled by bonding several spherical or circular particles [17]. Many papers have been published in the literature by using DEM in modelling and simulation of grinding mills, majority of them being limited in 2D. Hlungwani et al. [18] used a 2D laboratory ball mill to validate the DEM modeling of liner profile and mill speed effects. Cleary [19] used DEM to investigate charge behavior and power consumption in relation to operating conditions, liner geometry and charge composition in a 5m ball mill, also limited to the 2D code. Djordjevic et al. [20] have shown that 3D DEM simulations give more accurate results than 2D DEM simulation.

The DEM modeling that is used by many researchers [11], [13] involves determining the particles that are in contact, the amount of overlap and related velocities. Therefore the net forces acting on the contacting force can be obtained. Newton's second law of motion is applied to all particles to determine new particle positions, their velocities and acceleration.

Law of motion is based on the Newton second's law. In a particulate system, a single particle is affected by 3 types of forces: gravity, normal force, tangential forces and its motion can be described as :

$$m_i \frac{dv_i}{dt} = \sum F_{n,i} + F_{t,i} + g, \qquad (1)$$

The subscripts *i* is for representing particle, *v* is the velocity of the mass center, ω the angular velocity, $F_{n,i}$ the normal force of particle *i*, $F_{t,i}$ the tangential force of particle *i* and *g* is gravity. Taking the time step into consideration, the movement of a particle will be described by five factors: its position *x*, velocity*x'*, acceleration *x*["], angular velocity ω , angular acceleration ω' . They are all determined by the resultant force and resultant moment.

III. CONTACT MODEL

There are two major types of contact models: particleparticle and particle-geometry. The particle-particle contact is the focus in this research, which is mostly used for simulating different materials. Particle-particle contact models, can be classified as contacting force models and non-contacting force models. Generally four types of contact models are: continuous potential models, elastic model, visco-elastic and plastic models, in which the first one belongs to non-contacting force model, while the other three belong to the contacting force model. The continuous potential model is widely applied in molecular systems, which includes van der Waals forces, electrostatic forces, and liquid bridge forces. The elastic model can be sub-classified as linear elastic and nonlinear elastic models. In DEM, a collision can be either particle to particle or particle to geometry, and is represented by the contact model which show how the colliding bodies interact. Numerous contact models have been used to model the interaction of different particles. The linear spring-and-dashpot used by Cundall and Strack [7], Morrison and Cleary [21], and Datta and Rajamani [13]; the modified linear viscous damping model [22], the bilinear [22]; Hertz-Mindlin non-linear spring-and-dashpot, [17], [23]. Among all the contact model two model are commonly used.

A. Linear-Spring contact Model

The simplest model uses a linear assumption stating that the displacement is directly proportional to the force and based upon the work of Cundall and Strack. For collisions, the force is decomposed into normal and tangential forces with separate spring-dashpot elements, illustrated in Fig. 4. For each component, we define a spring and dashpot to calculate the force. The transition between a sticking and sliding collision is controlled by the coefficient of friction, which poses an upper limit to the tangential force. Here the characteristic impact (or overlap velocity) is a required input parameter [24].



Fig. 4. Linear-spring contact model [25]

In linear spring-and-dashpot contact model, the contact force in normal direction F_n can be governed by

$$F_n = -b_n V_n + k_n U_n, \tag{2}$$

where b_n, U_n, k_n, V_n are The normal damping constant, the overlap of contacting particles, the normal contact stiffness, and the relative normal velocity of particles.

$$F_s = F_s^{\ o} + k_s (U_s + U_s^{\ o}), \tag{3}$$

where F_s^{o} , U_s , k_s , U_s^{o} are the contact shear force during the previous time step, the relative tangential displacement, the shear contact stiffness, and the relative tangential displacement for the previous time step, Δt . The linear spring-and-dashpot model is widely used, especially in modeling particles in fluid, but for dry grinding it has one major problem: it is centrally to the law of physics as a model for particle collisions. In this model the viscous damping is assumed to be maximum as the particles are coming into contact and also as the particles are about to separate. This is not what is expected to happen. Damping should be a minimum when the particles first come into contact and also as the particles rebound. Due to the rather unphysical nature of the linear spring-and-dashpot model hypothesized by Sarracino et al. [22] may be adequate for charge motion and power draw predictions, but it wouldnt generate accurate impact energy spectrum predictions

B. Hertz-Mindlin Contact Model

The Hertz-Mindlin model shown in the Fig. 5 below, is the most commonly used within EDEM simulations [17]. The model uses a spring-dashpot response to normal contact between particles and/or geometry and a Coulomb friction coefficient μ for shear interactions and a second springdashpot response to tangential or rolling friction interaction. It provides an alternative, to the more common linear spring-anddashpot modeling. It illustrates more detailed and realistic the interaction between the two particles A and B than the springand-dashpot model [26]. Unlike the linear contact model, in the Hertz-Mindlin contact model the normal spring stiffness, k_n , varies according to the amount of overlap, U_n , between the contacting particles, in accordance with Hertzian contact theory developed by Hertz [27]. The total force between the particles can be divided into normal and tangential forces. Spring and damping components are available for both the forces, friction is available only for tangential component and coefficient of restitution is related to the normal force component. This model calculates the normal and tangential forces using material properties such as the coefficient of restitution, Youngs modulus, Poissons ratio, size and mass It is a non-linear elastic model and is thus well suited to the non-cohesive interactions which are to be used within the computational models [28].



Fig. 5. Hertz-Mindlin model [28]

Using Hertz-Mindlin contact model, the interaction of particles is governed by the Hertz theory

$$F_n = 2/3P_{max}\pi a^2,\tag{4}$$

where P_{max} is the maximum pressure at the point of contact and *a* is the area covered by the contacting bodies.

$$a = \frac{3P_{max}R^*}{4E^*},\tag{5}$$

 R^* and E^* are the reduced radius of contacting bodies and young modulus respectively. Hence the force- displacement relation in the normal direction is calculated by

$$F_n = -k_n U_n^{3/2},$$
 (6)

 U_n is the contact overlap and k_n is the normal contact stiffness.

IV. CONTACT MODEL PARAMETERS

Material properties and material interaction parameters, such as: the spring stiffness - or Youngs modulus and poisons ratio, damping constant or coefficient of restitution and coefficient of friction are required in the early discussed contact models. Chandramohan [29] said that instead of using estimate and approximate values of material interaction properties, they can rather be measured to provide an overall reliable and accurate result in predicting the motion of grinding media in ball mill.

A. Coefficient of Restitution

The damping constant and coefficient of restitution are important interaction parameters for the prediction of charge motion and energy distribution of particles inside the rotating mill. They represent measures of the energy that is lost during a collision. The coefficient of restitution is defined as the ratio of the relative velocities of colliding bodies just before contact, to the relative velocities just after the collision [27].

B. Contact Stiffness, Young's modulus and Poison's ratio

The resultant force from the overlap at the point of contact is a function of the contact stiffness. the selection of this parameter is necessary in DEM measurement. In Hertz-Mindlin model, the stiffness k is known as a function of young's modulus E and Poissons ration ν which are related such that

$$E = 2(1+\nu)G,\tag{7}$$

where G is the elastic shear modulus which is mostly used in DEM simulations.

C. Coefficients of static Friction

The coefficient of static friction is the friction force between two objects when neither of the objects is moving. The coefficient of kinetic friction is the force between two objects when one object is moving, or if two objects are moving against one another. The coefficient of friction governs the initialization of slip between two particles experiencing tangential interaction. Nierop et al. [30] reported the variation of the power draw with coefficient of friction, and suggested that The coefficient of friction may be an important parameter in DEM simulations.

V. OPERATIONAL PARAMETERS THAT AFFECT GRINDING MECHANISM OF BALL MILLS

In mineral grinding using ball mills, there are factors that have been investigated and applied in ball milling industries in order to maximize grinding efficiency [5].

A. Mill diameter

Bond [31] observed grinding efficiency as a function of ball mill diameter and established empirical formulas for recommended media and mill speed that take this factor into account. As well, mill with different length to diameter ration for a given power rating will yield different material retention times, the longer the units being utilized for a high reduction ratios and the shorter ones where over-grinding is of concern. Also related to both material and media rentation is discharged.

When designing a ball mill, much consideration is needed on the size of the drum of ball mill and the speed at which the drum is rotating. It has been shown by many researchers that the mill diameter and mill speed have a great impact on the grinding process of granular material [32], [33].

The net power consumption of a milling machine can be calculated using Equation. 8 below. This equation shows that the internal diameter of the milling machine has impact on the net power consumption.

$$NP = \lambda L D_m^{2.5} \tag{8}$$

where, NP is the net power, λ is the friction factor, L is the length of the drum, D_m is the internal mill diameter. Gupta



Fig. 6. Predicted Power draw for different diameter mills that have rectangular lifters at a mill speed of 60% CS and 50% mill charge [34]

Rowland also conducted a study and predicted the power draw and charge motion for different mills diameter with the same lifter shapes run at the same operating conditions. 50% ball load was selected according to the previous report that different diameter mills can draw maximum power around this load. Power draw for small scale to large scale mills was then predicted using the DEM simulation. Fig. 6, shows that the power draw for small diameter mills to large diameter mills that as mill diameter increases, simply mill power draw increases [34].

B. Mill speed

When designing a ball mill, much consideration is needed on the size of the drum of ball mill and the speed at which the drum is rotating. It has been shown by many researchers that the mill diameter and mill speed have a great impact on the grinding process of granular material [32], [33]. Deniz [33] investigated the effect of mill speed on the limestone and the clinker samples at batch grinding conditions based on a kinetic model. The effect of operational speed which is the fractional to critical speed φ_c on the grinding for model parameter a_T was found to be different for two different samples: $a_T = 0.0344exp(0.00301 \varphi_c)$ for clinker and $a_T = 0.0225 exp(0.06183 \varphi_c)$ for limestone. It was found that, for batch grinding, optimum grinding occurs at $\phi_c = 85\%$. Francioli [5] conducted another study on the effect of operational variables on ball milling. In order to study the effect of mill filling, powder filling, percentage of critical speed, ball size and percentage of solids, he carried out different tests. Grinding media of 25 mm, 30% mill filling, 100% powder filling and 75% of the critical speed was selected as the base condition and all other tests were varied according to the progress of the results and the need to evaluate tests with different operational variable. In this study he has analyzed the effect of mill speed, critical speed also known as the movement of the grinding media adjacent to mill shell during the entire mill rotation was taken as the reference speed. He concluded that ball mills can operate in two distinct regimes depending on the rotation speed: cascade and cataract, as illustrated in Fig. 7.



Fig. 7. Distinct regimes of rotation speed of a ball mill [5]

Cascade motion is more likely to result in breakage through attrition whereas cataract would favor collisions and, thus, body breakage. The effect caused by the variation of the critical speed can be seen in Fig. 8, which illustrate that the speed increases the center of mass of the charge inside the mill is dislocated towards the mill wall hence the increase of power consumption. However, when the speed gets closer to the critical speed the center of mass is dislocated to the mill center as the charge starts to centrifuge.



Fig. 8. influence of % of critical speed on power consumption in ball mill [5]

C. Mill filling charge

In grinding, it is also needed to know the rate at which the mill drum volume is occupied by, grinding media and ground material. Mill filling is the percentage of the mill volume occupied by the grinding media and the interstices between them [35]. This operational variable can be written as

$$J = \frac{Vgm}{V_m \times (1 - f_p)},\tag{9}$$

where, J, is the mill filling level,

Vgm, is the volume of the grinding media inside the mill, V_m , is the volume of the mill,

 f_p , is the fraction volume of interstices between the grinding media usually f_p has a value of 0.4.

The charge inside a mill can be given by:

$$f_c = \frac{Vma}{V_m(1 - f_p)},\tag{10}$$

where Vma is the volume of the material inside the mill [5]. The power consumption of a ball mill can also be calculated using Equation. 11 below

$$P = 2\pi T N, \tag{11}$$

where N is the rotational speed and T is the torque.

The torque necessary to maintain the offset in the center of gravity of the cascading charge from the rest position is given by:

$$T = M_b r_q \sin\alpha, \tag{12}$$

where M_b is the ball mass and r_g is the distance between the mill centre and α is the angle of repose of the ball charge.

The calculated power is maximum at about 50% ball load. According to Equation 12. Mill power is a function of ball mass (M_b) and the radius to the center of gravity of ball mass (r_g) . As the mill filling increases, the ball mass M_b increases but r_g decreases [13].



Fig. 9. Effect of mill filling on power consumption of ball mill [5]

Changing the mill filling can also affect the power consumption. Fig. 9, shows that more energy is required when there is an increase of the charge inside the mill as well as, the variation of the center of mass as the percentage of mill filling also plays a major role when it is changed [5].

D. Grinding time

Emami et al. [36] developed a model based on experimental observations to describe the effect of grinding time on the changes of surface area of a rock material during intensive grinding process. Validation and testing the model were performed experimentally using a natural chalcopyrite mineral. The conclusion was that, there is a variation of surface area with respect to time.

It has been reported that in some cases, specific surface area increases first with increasing grinding time but reaches a constant value after a certain grinding time. For such circumstances, Tanaka and Chodacov [36] have proposed the following equation to describe the process of new surface formation:

$$S = S_{max}(1 - e^{k_1 t})$$
(13)

S is the specific surface area at a given time t, and Smax is the maximum attainable specific surface area. The constant k_1 implies the significance of rate constant of the new surface formation.

E. Media size

Grinding media size and shape has a great impact on the grinding operation cost and results in huge consumption of liner and affect the overall performance of ball mill. The ball size in a mill has a significant influence on the mill throughput, power consumption and ground material size Austin et al. [37], Fuerstenau et al. [23]; [38]. The basic condition, which must be met while grinding the material in a mill is that the ball, while breaking the material grain, causes in it stress which is higher than the grain hardness Many researchers have worked on this problem trying to evaluate the effect of media size on the breakage rate material and on the power consumption of ball mill [38], [39].

Kabezya and Motjotji conducted an investigation to determine the effect of the ball diameter sizes on milling operation. A laboratory size ball mill was used with different ball media sizes of 10 mm, 20 mm, and 30 mm respectively. The material used to perform the experiment was Quartz arranged into 3 mono-sizes namely -8mm+5.6mm, -4mm+2.8mm, and -2 mm +1.4mm for the experiment. A mill run having a mixture of the 3 ball diameter sizes was also conducted. It was found that, the 30 mm diameter balls were most effective of the three sizes during the grinding of the 3 mono-size feed material samples. The 10 mm diameter balls were the least effective as minimum particle breakage was observed whereas the 20 mm diameter balls were relatively effective to some extent [40].

Magdalinovic also suggests that larger diameter balls have more energy whereas balls having smaller diameters have less energy. These different energies are however relative to the optimum ball diameter, which differs according to the size of the mill as well as the desired size reduction of the feed material [41], [42].

Kabezya and Motjotji also suggested that the mixture of different size of grinding ball can be used for more efficient ball mill. in their findings mixing the 3 different size of grinding ball showed that, the power draw for the ball combination mill run displays a decreasing trend and thus was the most efficiency with regards to the utilization of power towards particle breakage [40].

Kano et al [43], conducted another study by dry grinding gibbsite powder was in air using a tumbling ball mill with mono-size grinding media ranging from 4.8 to 31.7 mm diameter. The grinding device used in his work was a tumbling ball mill made of stainless steel, whose inner diameter, dM, was 121 mm and length, IM, was 142 mm. Steel mono-size balls, having different diameters, dB, of 4.8, 6.4, 7.9, 10.2, 12.7, 15.9, 19.1, 25.4, and 31.7 mm were prepared as grinding media. The ball-filling ratio, J, was kept constant at apparently 40% and the powder sample was charged at 20% for the mill volume. The mill was started running at the rotational speed, N, under dry atmospheric conditions, and the grinding time, t, was varied from 15 to 180 minutes. The main purpose was to investigate ball size effect on grinding rate. The grinding rate increases with an increase in the rotational speed of the mill, subsequently, it falls around the critical speed. The maximum grinding rate shifts toward higher rotational speed range as the ball size becomes large. Kano concluded that, the grinding rate is proportional to the specific impact energy regardless not only of the mill diameter and ball-filling ratio but also of the mono-size ball diameter.

F. Grinding media motion

In mineral grinding the media motion serves to hit the rock material and break it into small and fine materials. Grinding in ball mill has three distinct regime of rotation as seen in Fig. 7. According to Yi Sun et al [44], the motion state of practical charge (material and grinding media) is too complicated to be described precisely. Some researchers considered the grinding media to behave as a single grinding media, others considered the grinding media as the center of the entire mass which was considered as a rigid body, since these considerations ignored some other factors such as the size and shape of particles, it will surely cause variation between theoretical simulations and experiment.

Yi Sun et al [44], used discrete element method to simulate the motion of grinding media inside the drum of ball mill, which demonstrated that the grinding media motion generates the grinding effect in a cascading motion. The force analysis is crucial for the contact model to analyze media motion by considering grinding media as an individual smooth round sphere [44].

In conclusion, to achieve a great impact of the grinding media, it is required to increase the height of fall of the grinding media which will in turn increase the potential energy of the ball. Therefore much consideration of the mill filling level is needed because the mill filling volume rate has much effect on the motion and the contact impact of the grinding media [44].

G. Lifters and liner

Husni Usman [45] studied the effect of lifters configuration on the efficiency of the tumbling mill. The method used was simulation using Miitraj software. The study of effect of different lifter configuration and operating parameter on the mill efficiency and performance. The rail lifters was concluded to drew higher energy whereas High-Low lifter (Hi-Lo) improves the efficiency of the mill by approximately 22% [45]. For mills with lower mill charge and higher speed, the power usage for different lifters also showed a small difference. The energy requirements of High (Hi) lifter also varied slightly from the other lifters. Which shows that, the Hi lifter improves in the energy efficiency of the mill by approximately 6.7%. The throughput sizes of the different lifters become finer than at the higher load and lower speed. High (Hi) lifter demonstrated higher rate of breakage rates than the rail and Hi-Lo lifters . The Hi lifter at certain operating conditions would improve both the mill efficiency and breakage rate.

Another study conducted by Augustine B. Makokha and Michael H. Moys [46]–[48] has shown that the liner configuration and lifters has much impact on the breakage rate of the ground material. The study was about retrofitting worn liners with cone-lifters. Experiments were performed in a batch wise mode. In all conducted tests, the quartz material was ground for a total period of 4 min. Assessment of the performance of the three liner profiles under investigation and a comparison of the grinding data was made to which a conclusion was made that the liner profile significantly influences the milling rate and fines production in the mill. Fig. 10, also shows another study conducted by Yin et al, showing that the effect of lifter height has less significant effect on power draw of ball mill [49].



Fig. 10. Power draw at different lifter heights. [49]

VI. CONCLUSION AND RECOMMENDATION

From the literature substantial progress has been made in understanding and modeling the grinding process of grinding mills. With the availability of computer power and advanced numerical tools such as DEM grinding mechanism can be predicted using different methods. It was found that Hertz-Mindlin contact model is suitable for modeling the grinding process of a ball mill. It can be able to predict the behaviour of the charge inside the ball mill. It also need a high time step for better prediction. Contact model parameters also needs to be considered as they can affect results in DEM simulations. It was also found that for effective grinding, the media motion needs to be considered. The motion of the charge affect the power consumption of the grinding mill, the quality of throughput material, the wear of the liner and lifter geometry, and the overall efficiency of a grinding mill. Trajectory of grinding media depends on the mill filling level which determines the falling height of the material, the speed of the drum, and the type of liner used. Most of the research considered different parameters and ignored the lifter profile impact on the overall performance of ball mill. The Breakage of the material also was found to be affected by ball diameter and mill speed. The development of the past research done demands further research. Some of the key areas that must be targeted include:

- 1) First to focus on study of small scale ball mill by considering parameters that affect the grinding process of ball mill, mainly the liner geometry which can also affect the performance of ball mill. In the past researches the main focus has been on the effect of mill filling and grinding media on industrial ball mill with little or no attention on the effect of liner/lifter profile, lifters number, and lifters height, which also has a significant impact on the breakage rate and power consumption of a grinding mill.
- 2) It is also needed to consider the size distribution or a mixture of ball diameter in a ball mill. Many researchers have shown that bigger grinding balls have more energy whereas balls having smaller diameters have less energy, none has shown the correlation of the grinding ball size with the ground material size. Kano has said that, the grinding rate is proportional to the specific impact energy regardless not only of the mill diameter and ball-filling ratio but also of the mono-size ball diameter. Since there is a contradiction about the effect of ball size or ball diameter on the breakage rate parameter, a deep study is required to establish the correlation of ball size and breakage of the material. Therefore the optimum mixture of balls to mill a given type of charge material will be established.
- 3) The relationship between: Mill speed, ball size distribution, and lifter geometry, has not yet been clarified. It is not obvious which is dominant in power consumption of a grinding mill. Therefore it is needed to investigate the correlation between mill speed, ball size distribution, and lifters geometry.

References

REFERENCES

- D. Saramak, T. Tumidajski, M. Brozek, T. Gawenda, and Z. Naziemiec, "Aspects of comminution flowsheets design in processing of mineral raw materials," *Gospod. Surowcami Miner. / Miner. Resour. Manag.*, vol. 26, no. 4, pp. 59–69, 2010.
- [2] D. o. E. US, "Mining Industry energy bandwidth study," pp. 22–30, 2007.
- [3] P. Khadka, J. Ro, H. Kim, I. Kim, J. T. Kim, H. Kim, J. M. Cho, G. Yun, and J. Lee, "Pharmaceutical particle technologies: An approach to improve drug solubility, dissolution and bioavailability," *asian journal of pharmaceutical sciences*, vol. 9, no. 6, pp. 304–316, 2014.
- [4] K. Asano, H. Enoki, and E. Akiba, "Synthesis of HCP, FCC and BCC structure alloys in the Mg-Ti binary system by means of ball milling," *Journal of Alloys and Compounds*, vol. 480, no. 2, pp. 558–563, 2009.
- [5] D. M. Francioli, "Effect of Operational Variables on Ball Milling," *Escola Politecnica*, 2015.
- [6] B. Mishra, "A review of computer simulation of tumbling mills by the discrete element method: part icontact mechanics," *International journal of mineral processing*, vol. 71, no. 1-4, pp. 73–93, 2003.
- [7] P. A. Cundall and O. D. L. Strack, "A discrete numerical model for granular assemblies," *Géotechnique*, vol. 29, no. 1, pp. 47– 65, 1979.
- [8] L. Lorig and B. Brady, "13 a hybrid computational scheme for excavation and support design in jointed rock media," in *Design and Performance of Underground Excavations: ISRM SymposiumCambridge, UK, 3–6 September 1984.* Thomas Telford Publishing, 1984, pp. 105–112.
- [9] C. S. Campbell and C. E. Brennen, "Computer simulations of granular shear flows," J. Fluid Mech., vol. 151, pp. 167–188, 1985.
- [10] J. M. Ti and B. T. Corkum, "Strength behavior of granular materials using discrete numerical modelling," 1988.
- [11] B. K. Mishra and R. K. Rajamani, "The discrete element method for the simulation of ball mills," vol. 16, no. August 1991, pp. 598–604, 1992.
- [12] J. Raasch, "Trajectories and Impact Velocities of Grinding Bodies in Planetary Ball Mills," *Chem. Eng. Technol.*, vol. 212, no. 1, pp. 245–253, 1992.
- [13] A. Datta, B. K. Mishra, and R. K. Rajamani, "Analysis of power draw in ball mills by the discrete element method," *Can. Metall. Q.*, vol. 38, no. 2, pp. 133–140, 1999.
- [14] S. Kim and W. S. Choi, "Analysis of ball movement for research of grinding mechanism of a stirred ball mill with 3D discrete element method," *Korean Journal of Chemical Engineering*, vol. 25, no. 3, pp. 585–592, 2008. [Online]. Available: http://dx.doi.org/10.1007/s11814-008-0099-x
- [15] J. M. Ting, "A robust algorithm for ellipse-based discrete element modelling of granular materials," *Computers and Geotechnics*, vol. 13, no. 3, pp. 175–186, 1992.
- [16] J. Ghaboussi and R. Barbosa, "Three-dimensional discrete element method for granular materials," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 14, no. 7, pp. 451–472, 1990.
- [17] DEM Solutions, "EDEM 2.4 User Guide," pp. 1-134, 2011.
- [18] O. Hlungwani, J. Rikhotso, H. Dong, and M. Moys, "Further validation of dem modeling of milling: effects of liner profile and mill speed," *Minerals Engineering*, vol. 16, no. 10, pp. 993– 998, 2003.
- [19] P. W. Cleary, "Charge behaviour and power consumption in ball mills: sensitivity to mill operating conditions, liner geometry and charge composition," *International journal of mineral processing*, vol. 63, no. 2, pp. 79–114, 2001.
- [20] N. Djordjevic, F. Shi, and R. Morrison, "Determination of lifter design, speed and filling effects in ag mills by 3d dem," *Minerals Engineering*, vol. 17, no. 11-12, pp. 1135–1142, 2004.
- [21] R. D. Morrison and P. W. Cleary, "Using DEM to model ore breakage within a pilot scale SAG mill," *Minerals Engineering*, vol. 17, no. 11-12, pp. 1117–1124, 2004.

- [22] R. Sarracino, A. McBride, and M. Powell, "Using particle flow code to investigate energy dissipation in a rotary grinding mill," in Numerical Modeling in Micromechanics via Particle Methods-2004: Proceedings of the 2nd International PFC Symposium, Kyoto, Japan, 28-29 October 2004. CRC Press, 2004, p. 111.
- [23] D. W. Fuerstenau and J. J. Lutch, "The effect of ball size on the energy efficiency of hybrid high-pressure roll mill r ball mill grinding," pp. 199–204, 1999.
- [24] C. Kulya, "Using Discrete Element Modelling (DEM) and Breakage Experiments To Model The Comminution Action in a Tumbling Mill," Master thesis, University of Cape Town, 2008.
- [25] H. Maghsoodi and E. Luijten, "Chaotic dynamics in a slowly rotating drum," *Revista Cubana de Ftsica*, vol. 33, no. 1, pp. 50–54, 2016.
- [26] J. Jaeger, New solutions in contact mechanics. Wit Pr/Computational Mechanics, 2005.
- [27] K. Johnson, "Contact mechanics, cambridge university press, cambridge."," 1985.
- [28] J. Härtl and J. Y. Ooi, "Experiments and simulations of direct shear tests: porosity, contact friction and bulk friction," *Granular Matter*, vol. 10, no. 4, p. 263, 2008.
- [29] R. Chandramohan, "Measurement of particle interaction properties for the incorporation into discrete element methods," Ph.D. dissertation, University of Cape Town, 2005.
- [30] M. Van Nierop, G. Glover, A. Hinde, and M. Moys, "A discrete element method investigation of the charge motion and power draw of an experimental two-dimensional mill," *International Journal of Mineral Processing*, vol. 61, no. 2, pp. 77–92, 2001.
- [31] F. C. Bond, "Crushing and grinding calculations, part i," Br. Chem. Eng., vol. 6, pp. 378–385, 1961.
- [32] R. P. King, "Technical Notes 8 Grinding," Media, 2000.
- [33] V. Deniz, "The effect of mill speed on kinetic breakage parameters of clinker and limestone," vol. 34, pp. 1365–1371, 2004.
- [34] C. A. Rowland, "Bonds method for selection of ball mills," Advances in Comminution, pp. 385–397, 2006.
- [35] K. K. Kiangi and M. H. Moys, "Particle filling and size effects on the ball load behaviour and power in a dry pilot mill: Experimental study," *Powder Technology*, vol. 187, no. 1, pp. 79–87, 2008.
- [36] A. Emami, M. S. Bafghi, J. Vahdati Khaki, and A. Zakeri, "The effect of grinding time on the specific surface area during intensive grinding of mineral powders," *Iranian Journal of Materials science and Engineering*, vol. 6, 2009.
- [37] L. Austin, K. Shoji, and P. T. Luckie, "The effect of ball size on mill performance," *Powder Technology*, vol. 14, no. 1, pp. 71–79, 1976.
- [38] N. Kotake, K. Daibo, T. Yamamoto, and Y. Kanda, "Experimental investigation on a grinding rate constant of solid materials by a ball mill effect of ball diameter and feed size," vol. 144, pp. 196–203, 2004.
- [39] H. Ipek, "Effect of grinding media shapes on breakage parameters," *Particle and Particle Systems Characterization*, vol. 24, no. 3, pp. 229–235, 2007.
- [40] K. Km and H. Motjotji, "Material Science & Engineering The Effect of Ball Size Diameter on Milling Performance," vol. 4, no. 1, pp. 4–6, 2015.
- [41] N. Nistlaba and S. Lameck, "Effects of Grinding Media Shapes on Ball Mill Performance," Masters thesis, University of the Witwatersrand, Johannesburg, 2005.
- [42] N. Magdalinovic, M. Trumic, M. Trumic, and L. Andric, "The optimal ball diameter in a mill," *Physicochem. Probl. Miner. Process.*, vol. 48, no. 2, pp. 329–339, 2012.
- [43] J. Kano, H. Mio, F. Saito, and M. Miyazaki, "Correlation of grinding rate of gibbsite with impact energy in tumbling mill with mono-size balls," *Minerals engineering*, vol. 14, no. 10, pp. 1213–1223, 2001.
- [44] Y. I. Sun, M. Dong, Y. Mao, and D. Fan, "Analysis on Grinding media Motion in Ball Mill by Discrete Element Method," in *1st Int. Conf. Manuf. Eng. Qual. Prod. Syst.*, vol. I, no. Volume I, pp. 227–231.
- [45] H. Usman, "Measuring the efficiency of the tumbling mill as a function of lifter configuration and operating parameters."
- [46] A. B. Makokha and M. H. Moys, "Towards optimising ballmilling capacity : Effect of lifter design," *Minerals Engineering*, vol. 19, pp. 1439–1445, 2006.

- [47] A. B. Makokha, M. H. Moys, M. M. Bwalya, and K. Kimera, "A new approach to optimising the life and performance of worn liners in ball mills : Experimental study and DEM simulation," vol. 84, pp. 221–227, 2007.
- [48] A. B. Makokha, M. H. Moys, and M. M. Bwalya, "Modeling the RTD of an industrial overflow ball mill as a function of load volume and slurry concentration," *Minerals Engineering*, vol. 24, no. 3-4, pp. 335–340, 2011. [Online]. Available: http://dx.doi.org/10.1016/j.mineng.2010.11.001
- [49] Z. Yin, Y. Peng, Z. Zhu, Z. Yu, and T. Li, "Impact load behavior between different charge and lifter in a laboratory-scale mill," *Materials*, vol. 10, no. 8, 2017.