MAGNETOHYDRODYNAMIC FREE CONVENTION FLOW OF A HEAT GENERATING FLUID PAST A SEMI-INFINITE VERTICAL POROUS PLATE WITH VARIABLE SUCTION

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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This thesis has been submitted for examination with our approval as University Supervisors.

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DEDICATION

I would like to dedicate this thesis to my wife Regina Mueni, and my children Daniel Mwasa, Shadrack Mutua and Francis Musyoki for the love, affection, encouragement and moral support they offered to me throughout the entire course. I also, dedicate this thesis to my both late mum and dad Anne Mulimi Nzambali and Musyoki Mumo for their fundamental inspiration and influence in early part of my life.

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ABBREVIATIONS

MHD	Magnetohydrodynamics
FDM	Finite Différence Method

NOMENCLATURE

ROMAN SYMBOL	QUANTITY
C_p	Specific heat, Jkg $^{-1}$ K^{-1}
D	Diffusion coefficient, m^2 s^{-1}
E	Electric field, Volt m^{-1}
E	Electronic charge, C m^{-3}
g	due to gravity, ms^{-2}
Gr	Grashof number
Gc	Modified Grashof number
H	Magnetic field intensity, Wb m^{-2}
J	Current density, Am^{-2}
J_x+, j_y+, j_z+	Components of current density, Am^{-2}
K	Thermal conductivity, W m^{-1} K^{-1}
M	Hall parameter
M^2	Magnetic parameter
Nu	Nusselt number
u	Primary Velocity ms^{-1}
v	secondary velocity ms^{-1}
Ec	Eckert number
Er	Rotational Parameter
S	Suction Parameter

w	Injection Parameter
t	Time, s
P_r	Prandtl number
Q^+	Internal heat generation, W m^{-3}
Q^+_1	Coefficient of proportionality absorption of radiation, W m^{-3} K^{-1}
Sc	Schmidt number
T	Temperature of the fluid, K

GREEK SYMBOLS	QUANTITY
β^*	Coefficient of expansion due to concentration gradients, K^{-1}
θ	Dimensionless temperature of fluid
ρ	Fluid density, kg m^{-3}
υ	Kinematics viscosity, m^2 s^{-1}
□	Heat source parameter
μ_e	Magnetic permeability, H m^{-1}
ω_e	Cyclotron frequency, Hz
η_e	Number density of electrons
□	Electrical conductivity, Ω^{-1} m^{-1}
τ_e	Collision time of electrons, s
τ_x	Skin friction due to primary velocity profile
τ_y	Skin friction due to secondary velocity profile
e	electric charge, A
Ω	Angular Velocity
μ	Dynamic Viscosity, $NSMS^{-2}$

ABSTRACT

In this study, a magnetohydrodynamic convection flow of an electrically conducting heat generating fluid past a semi-infinite vertical porous plate with variable suction is considered. The fluid flow is unsteady and a variable magnetic field is transversely applied to the plate. Evaluation of velocity gradients, temperature gradients and concentration gradients across the plate is done. Observations and discussions of the effects of various parameters on flow variables is done. The non-dimensional parameters observed and discussed are Hall parameter, M; Magnetic number, M^2; Eckert number, Ec; Rotational parameter, Er; Suction parameter, S and Injection parameter, w.

The velocity profiles, temperature profiles and concentration profiles are presented graphically for both convectional heating and free convectional cooling of the plate. The skin friction and rate of heat transfer values are obtained and presented in tables.

For free convectional heating and cooling of the plate, the Grashof number is taken as constants -5 and 5 respectively. Prandtl number is 0.71 which corresponds to air. The variation of the parameters mentioned above is noted to increase or decrease or had no effect on the skin friction, mass transfer, rate of heat transfer, the velocity profiles, concentration profiles and temperature profiles.

CHAPTER ONE

INTRODUCTION AND LITERATURE REVIEW

1.0 INTRODUCTION

There are three classes of matter, that is, solid, liquid and gas. Liquids or gases are termed as fluids. A solid is a matter in which the distance between its molecules does not change when a force is applied on it. A fluid (liquid or gas) is the matter in which the distance between its molecules changes on application of some force. Fluids undergo deformation whenever forces applied on but the mass remains constant. There are two types of fluids, that is, Newtonian fluids where viscosity does not change with the rate of deformation and non-Newtonian fluids where viscosity varies with the rate of deformation.

Fluid mechanics is a broad branch of science and mathematics which deals with dynamics of liquids and gases. Fluid mechanics is divided into three branches, that is, fluid kinematics, fluid statics and fluid dynamics. Fluid kinematics exists when fluid is in motion and displacement, velocities and acceleration play a significant role. Fluid statics is the study of fluids that are stationary or at rest.

Fluid dynamics is the branch of science which deals with forces that causes the motion of fluids. Some types of fluid flows are steady flow and unsteady flow, uniform flow and non-uniform flow (or varied flow), laminar flow and turbulent flow and lastly subcritical flow, critical flow and supercritical flow. The flow could also be rotational or irrotational. A steady flow is a flow where density, pressure, velocity including other thermodynamic properties are independent of time, that is, $\frac{\partial p}{\partial t} = 0$ where p may be velocity, density, pressure, temperature, etc. In unsteady flow $\frac{\partial p}{\partial t} \neq 0$ and in this case condition and properties associated with the fluid motion vary with time. In uniform flow, the fluid particles have same velocity at each point along the flow region. If there exist different velocities at different points of a particular section of flow field, this type of flow is a non-uniform flow.

A flow is said to be laminar flow if its Reynolds number in less than 500 and if it is more than 2000 then the flow is turbulent, but the transitional flow is where the Reynolds number is between 500 and 2000. Laminar flow is orderly and there is no mixing between different fluid layers (except by molecular motion which is negligible).

The turbulent flow is characterized by strong mixing of particles in the direction normal to the boundary. The main flow is superimposed by subsidiary motion at right angles to it and thus there is creation of transverse mixing of fluid particles. On the other hand, the fluid in laminar flow appears to have layers which move smoothly and orderly in defined paths as opposed to the case of turbulent flow where fluid has irregular fluctuations of velocity at every point.

When the flow is mainly influenced by gravitational force we obtain three types of flows, that is, subcritical flow, critical flow and supercritical flow and in this case, Froude number (Fr) is used to determine the type of flow. So if $Fr<1$ the flow is subcritical, Fr=1 the flow is critical and Fr>1, the flow is supercritical (or rapid or shooting or torrential). Lastly we have Barotropic flow in which the pressure is a function of the density.

The fluid flow depends on the geometry of the surface the fluid is flowing on. For example, flow in pipes and channels are controlled by geometry of cross-section, surface roughness and velocity distribution. In channels the flow is due to gravity force while in pipes it takes place at the expense of hydraulic pressure where the pressure decreases in the direction of the flow. For geometrical cross-sections, pipes are generally circular while in open channels they could be triangular, rectangular, trapezoidal, parabolic, elliptic etc. As such the maximum velocity in pipes is at the centre while for channels it is just below the surface of the fluid. The velocity decrease towards the surface of the container.

In fluid flow, fluid currents which could be regular or irregular could equalize temperature in the entire fluid. There are two types of convection heat transfer. These are free (or natural) and forced convection. Free convection flow takes place when density varies due to concentration and temperature gradients. In forced convection heat transfer is due external forces. Conduction takes place when there exist free or partially free electrons (quasi-free electrons) which can be moved under the action of magnetic fields or electric fields. In solids, electrons are bound but can move for a restricted and specific distance before collision occurs within the crystal lattice. Dynamical effects such as conduction and Hall effects are observed when fields are applied to the solid conduction, though the motion of mass does not occur in general. The effect of the applied magnetic fields on the atoms translates into stresses in the structure of the lattice. In fluids, there exist both electrons and ionized particles and when magnetic fields are imposed on the fluid, dynamical effects are very significant and interestingly the fluid itself will move, resulting in bulk motion of the medium (fluid). The motion of the medium (fluid) alters the magnetic and electric field magnitudes through their interaction. The motion of the fluid do alter the strength and direction of electromagnetic fields in terms of strength and direction.

The electrons are accelerated by the applied (magnetic) fields but, their directions are changed by collisions, so that the motion of electrons are opposed by an effective frictional force (= $\omega m v$), where ω is an effective collision frequency; m is the mass of an electron and v is the velocity of the electrons. Units of ω is s^{-1}, m is kg and v is m/s and hence unit of $\omega m v$ is Newton, which is the unit of force. Ohm's law gives a balance of applied magnetic force from fields and effective frictional force on the electrons (the frictional drag). When frequency of the applied magnetic fields are comparable to effective collision frequency (of electrons), the electrons accelerate and decelerate between the collisions, and thus the inertial effects occurs and the conductivity becomes very complex. If the frequencies of the magnetic fields are above the collision frequency, the electrons and ions are accelerated in the opposite directions by electric fields and tend to separate and this creates strong electrostatic restoring forces which creates oscillation in the charge density. These oscillations are called plasma oscillation and exist only when frequency of the magnetic field is higher than frequency of the electron collisions. If the frequency of the fields is lower than collision frequency, there is no charge separation and the name of these lower frequencies is magnetohydrodynamics waves.

In conducting fluids, collision frequency is well above field frequency and for extremely good fluid conductors then there is a wider frequency range and thus the Ohm's law becomes valid to be applied. The electrons and ions move together without being separated because collision frequency is far greater than frequency of the fields. Now, since electrons and ions move together, electric fields arise due to the motion of the fluid which ignites a current flow created by magnetic fields which are varying with time or charge distributions external to the fluid. The mechanical motion of the system can now be described in terms of a single conducting fluid using hydrodynamic variables of density, pressure and velocity. At low frequency of magnetic fields, displacement current in Ampere's Law is neglected and this means the mechanical motion of the system will be expressed without displacement current. Thus, the mechanical motion of the system is an approximation without displacement current and this approximation is called magnetohydrodynamics.

In magnetohydrodynamic there is very dense ionized gases and the collision frequencies are extremely high compared to the frequencies of the magnetic fields, but, in less dense ionized gases the collision frequency is far smaller compared to the frequency of the field, though, in these low dense ionized gases may exist low frequency domain where magnetohydrodynamic equations are applicable to quasi stationary processes. In astrophysical applications we do not neglect charge separation, displacement current and inertial effects of electron and ions are not neglected as done in Magnetohydrodynamics. Thus, the separate inertial effects of the electrons and ions must be included in the description of the motion and this is the discipline called plasma physics.

1.1.1 THE HALL EFFECT

The current in metal is carried by electrons. If a conductor is in magnetic field, such that the direction of flow of current is at right angle to the magnetic field, a voltage is developed across the conductor in the direction perpendicular to both current and field. The phenomenon by which such voltage is developed is called Hall Effect and voltage itself is Hall voltage.

1.1.2 HYDROMAGNETIC

Dynamics is the movement of an object due to the applied forces while hydrodynamics is movement of fluid when forces are applied. Electromagnetism is the study of the interaction between electric fields and magnetic fields. Thus, the definition of hydromagnetics is the study of interaction between hydrodynamics (fluid dynamics) and electromagnetism.

1.1.3 FREE CONVECTION FLOW

Due various applications in engineering and universe, MHD free convection flow has become significant. A fluid flow in which the motion is as a result of body force acting on the fluid in which there are density gradients is called a free convection flow. Temperature or concentration gradients existing in the fluid yields to density gradients while the gravitational force yields to the body force. Thus the action of the body force on the fluid amounts to buoyancy that eventually induces free convection current.

1.1.4 FORCED CONVECTION FLOW

The forced convention flow is the flow in which the buoyancy force is insignificant while the velocity of the main stream is significant.

1.1.5 HEAT TRANSFER

Heat transfer is basically due to temperature difference in a body or different bodies. Heat transfer takes place in three modes, that is, conduction, convention and radiation (only at higher temperatures). If the fluid motion is due to buoyancy effects, which are due to density variation caused by the temperature difference in the fluid, the heat transfer is called free or natural convention. Radiation is the type of heat transfer which occurs at high temperatures of a body and can be emitted from solids, or liquids or gases and can also be emitted through vacuum. This investigation shall consider free convention heat transfer which can be caused by temperature differences or concentration gradients.

1.1.6 MASS TRANSFER

The relative motion of a mixture's species as a result of concentration gradients is termed as mass transfer. Thus, mass transfer is caused by concentration difference of the species in a mixture. Modes of heat transfer that are similar to convection and conduction do exist.

Mass transfer by free convection is studied in this study. In natural convection, external forces are not required since effects of buoyancy and the force of gravity induce the motion thereby resulting in the heat transfer. Thus both heat transfer and fluid flow due to convection rely on the fundamental principles of heat transfer and fluid flow. Significant laws of convection includes: conservation of mass, momentum conservation and energy conservation law.

1.1.7 BOUNDARY LAYER

When a real fluid (viscous fluid) flows on a stationary solid boundary, a layer of fluid which comes in contact with the boundary surface, adheres to it (on account of viscosity) and condition of "no slip" occurs (The "no slip" condition means that the velocity of fluid at a solid boundary must be same as that of the boundary itself). Thus, if the layer of fluid which cannot slip away from the boundary surface undergoes retardation, this retarded layer further causes retardation for the adjacent layers of the fluid. Hence there is a development of a small region around the boundary surface where the velocity of the flowing fluid increases rapidly from zero at the boundary surface and approaches the velocity of main stream. The layer adjacent to the boundary is known as boundary layer.

Boundary layer is formed whenever there is relative motion between the boundary and the fluid. Viscous shear stress takes place at the boundary layer.

In any fluid flow over a surface, there exist three boundary layers that is, velocity, thermal and concentration boundary layers. When velocity is zero and then increases to maximum this region is called velocity boundary layer. If fluid is in contact with isothermal plate (heated plate) the fluid in contact with the plate attain isothermal equilibrium at the plate's surface temperature. The particles on the plate pass heat energy to the adjoining fluid layers and a temperature gradient is developed. This thermal region is called thermal boundary layer. If there exist concentration gradients in the fluid between the plate and the free stream, then we have concentration boundary layer. We shall consider laminar flow to investigate convection mass transfer, convection heat transfer and surface friction.

1.1.8 MAGNETOHYDRODYNAMICS

The word magnetohydrodynamics consists of the words magneto-meaning magnetic, hydro-meaning water (or liquid) and dynamics meaning the motion of an object under influence of forces. Magnetohydrodynamics occurs when effective collision frequency of electrons is far more than the frequency of the applied fields and also where inertial effects and displacement currents are neglected. If frequencies of applied fields are far higher than effective collision frequency the displacement current, separate electrons from ions and inertial effects are considered (this is plasma physics). Then briefly, we can define magnetohydrodynamics as study of fluid flow in the presence of magnetic field where an induced electric field is created. MHD can also be defined as the physical mathematical framework where equations of the electromagnetic fields and dynamics are coupled. Thus, in MHD the currents induced in the matter (fluid) modify the field which has created it (electromagnetic fields). In MHD displacement current is neglected. Lastly, the significance of MHD theory is that conductive fluids can support magnetic fields where the presence of magnetic fields leads to forces that in turn get on the fluid thereby potentially altering the geometry and the strength of the magnetic fields themselves.

1.2 LITERATURE REVIEW

Faraday (1832) performed an experiment with mercury flowing in a glass tube placed between poles of a magnet; he discovered that a voltage was induced across the magnetic field perpendicular to both the direction of the flow and the magnetic field. In 1938, Hartman discussed both experimentally and theoretically the hydromagnetic flow between two parallel plates. But Alfuen in 1942 established transverse waves in electrically conducting fluids and explained its applications in astrophysics. Thus he deduced that electromagnetic theory and fluid dynamics interact resulting to hydromagnetics.

The interaction between the two branches (i.e. electromagnetic and fluid dynamics) which resulted to non-dimensional numbers was introduced by Linguistics in 1952. The non dimensional number $\sqrt{}BL(\sigma\mu_{e/\rho}\ )^{1/2}$, where B is the magnetic field, L is the characteristic length, σ is the electrical conductivity, μ_e is the magnetic permeability and ρ is the fluid density was found to be significant.

Chaturverdi (1996) studied MHD flow past an infinite porous plate with variables suction. Jha and Prasad (1992) investigated MHD free convection and Mass transfer flow through a porous medium with heat source. Ram *et al.* (1990) studied Heat and Mass transfer of viscous heat generating fluid with Hall currents. Takhar *et al.*. (1995) investigated the hydromagnetic convective flow of a heat generating fluid past a vertical plate with Hall current and heat flux through a porous medium. Kinyanjui *et al.* (2000) investigated MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption. Kwanza *et al.* (2003) Studied MHD Stokes free convection flow past an infinite vertical porous plate subjected to constant heat flux with ion-slip current and radiation absorption. Hydromagnetic flow past parallel porous plate was studied by Kafousia (1985) and Dorch (2007) explained the effect of magnetic field on a rotating porous plate.

Chartuverdi (1996) investigated the flow of an incompressible viscous fluid past an impulsively started horizontal plate and Magnetohydrodynamics flow past an infinite plate with a constant and variable suction. He also studied the finite difference of MHD stokes problem for a vertical infinite plate in a dissipative heat generating fluid with Hall and Ion-slip current. Kinyanjui *et al.* (1998) studied the MHD stokes problem for a vertical infinite plate in a dissipative rotating fluid with Hall current and they later investigated the effect of both Hall and Ion-slip currents on the flow of heat generating rotating fluid system. They observed that for an Eckert value of 0.02, there was a decrease in the primary velocity profile with an increase in Rotational parameter. In the case of secondary velocity profiles there is initially a decrease with an increase in Rotational parameter and as the distance from the plate increases, the secondary velocity profile increased. They also observed that an increase in Hall parameter has no effect on the temperature profile but an increase in times causes an increase in the temperature profiles.

Polhhausen (1921) studied the free convention flow past a semi-infinite vertical plate by the momentum integral method. But the similarity solution to free convection flow past a semi-infinite vertical plate was first presented by Ostrach (1953) who solved the non-linear coupled ordinary differential equations numerically on a computer. The fluid considered was air.

Kinyanjui *et al.* (1998) studied the finite difference analysis of free convection effects on MHD problem for a vertical plate in a dissipative rotating fluid system with constant heat flux and Hall current. Kinyanjui *et al.* (1999) also did a finite difference analysis of MHD stokes problem for a vertical infinite plate in a dissipative fluid with constant heat and Hall current. Takhar and Soundalgekar (1976) did a study on the viscous dissipation effects on heat transfer in boundary layer flow past a semi infinite horizontal flat plate.

Takhar and Soundalgekar (1972) studied a forced and free convective flow past a semi-infinite vertical plate and also (1977b) the MHD and heat transfer over a semi-infinite plate under a transverse magnetic field. Some of their other works include MHD free convection past a semi-infinite vertical plate with suction and injection. MHD free convection flow past a vertical semi-infinite plate with a uniform free –stream in 1985 and MHD free convection flow past a semi-infinite plate with uniform heat flux also in 1985.

Takhar, *et al.* (2000) did a study on the effects of radiation on the free convection flow past a semi-infinite vertical plate with mass transfer. They observed that both the temperature and velocity decreased as the distance from the plate's leading edge increased and that an increase in radiation parameter leads to a decrease in the temperature and velocity. Ram *et al.* (1998) solved the MHD stokes problem of a convective flow past a vertical infinite plate in a rotating fluid. They investigated the problem of hydro magnetic free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux and he (1990) solved the MHD stokes problem for a vertical infinite plate with Hall and ion-slip currents by explicit finite difference method. He observed that an increase in the Hall parameter leads to an increase in primary velocity profiles and it lead to a decrease in the secondary profiles. He also noted that an increase in hall parameter or Ion slip parameter lead to an increase in the temperature profiles.

Soundalgekar *et al.* (1985) used finite difference method to investigate free convection effects on Stokes problem for a vertical plate in a dissipative fluid with constant heat flux and he (1976) also studied free convection effects on the stokes problem for an infinite vertical plate. They also studied the free convection effects on MHD stokes problem for a vertical plate. Jha and Prasad (1992) investigated MHD free convection and mass transfer through a porous medium with heat source.

Kafousia *et al.* (1981) studied unsteady hydro magnetic layer flow past a non-conducting infinite vertical porous plate in the presence of a transverse magnetic field taking into account the effect of heat source on the free convection flow. He investigated the heat transfer in viscous incompressible and electrically conducting fluid. Reddy (1964) discussed the fluctuating MHD flow past an infinite porous flat plate by introducing the slip flow boundary condition. Sterwatson (1951) did a study on the flow of an incompressible viscous fluid past an impulsively started semi-infinite horizontal plate.

Takhar *et al.* (1980) investigated the hydro magnetic flow of a heat generating fluid past a vertical plate with Hall current and heat flux through a porous medium. Agrawal *et al.* (1983) studied the effect of Hall current on steady hydro magnetic free convective flow past an infinite vertical porous plate in a rotating viscous fluid system.

Despite the intensive investigation done on various areas of MHD by the above mentioned scientists and mathematicians, little attention has been directed to my main objective of this study. Not much has been done in the MHD free convection heat and

Mass transfer problem for a heat generating fluid past a semi – infinite vertical porous plate with variable suction.

1.3 STATEMENT OF THE RESEARCH PROBLEM

When an unsteady, heat generating electrically conducting fluid flows past a semi-infinite vertical porous plate with variable suction; velocity, temperature, rates of mass heat transfer, concentration and skin friction are affected.

The intention in this study is to obtain an approximate velocity profiles, temperature profiles, and concentration profiles of a heat generating fluid past a semi-infinite vertical porous plate with variable suction. This study, the magnetic field is applied transversely to the direction of the flow and suction is variable, see figure 2.1.

1.4 JUSTIFICATION

MHD free convection flow past a semi-infinite vertical porous plate with variable suction is a study which has many applications such as in MHD pumps, MHD power generator, purification of crude oil in petroleum industries, polymer technology and aerodynamic heating and accelerators. In the study of MHD free convection flow of a heating generating fluid past a semi-infinite vertical porous plate with variable suction finds very many applications in cooling of electronic devices (e.g. mobiles, computers etc.) and solar panels. Some of other applications in this study are design of; flow meters, MHD generators, heat exchangers, space vehicle, propulsion and breaking, electromagnetic pumps and MHD electrical power generation. Fluid flow involving rotation is observed in earth's atmosphere and in oceans. Meteorologist can use this study to understand dynamics of meteorology and air pollution. It is in the light of this that this study will be useful to welfare of mankind. The general governing equations used are momentum equation, Energy conservation equation, Concentration equation and Induction equation.

1.5 OBJECTIVES OF THE STUDY

1) To investigate the effect of various parameters on the flow variables in the flow of heat generating electrically conducting fluid past a semi – infinite vertical porous plate in the presence of a variable suction
2) To analyze the skin friction, rate of heat transfer and mass transfer
3) To determine the velocity and temperature profiles and concentration profiles.
4) To investigate the effect of mass diffusion (mass transfer), Hall current (induced current) and heat source parameters of a heat generating fluid past a semi – infinite vertical porous plate with variable (inhomogeneous) suction.

CHAPTER TWO

GOVERNING EQUATIONS

2.0 INTRODUCTION

The basic conservation laws are mass conservation, momentum conservation and energy conservation. These conservation laws must be satisfied fully to obtain the governing equations which are used to analyze the fluid flow. In order to analyze MHD free convection heat and mass transfer of a Heat generating fluid past a semi-infinite vertical porous plate with variable suction, the equations of continuity (conservation of mass), equation of motion, energy equation, induction equation (which obtained both from Maxwell's equations and Ohm's law equations). Numerical Techniques (finite difference method) is used to obtain solution of the systems of equations derived. The governing equations in dimensional form are by using none non-dimensionalised by using non-dimensional parameters.

2.1 CONSERVATION EQUATIONS

2.1.1 Mass conservation equation

Equation of mass conservation is also known as the equation of continuity and applies to all fluids which have no chemical reactions and also variations of all fluid properties are completely ignored except density since it influenced by gravitational force (ρg)

The equation of the conservation of mass is written as

$$\frac{D\rho}{Dt}+\rho\nabla.\vec{q}=0 \qquad 2.1$$

Since the density is constant, then this reduces to

$$\frac{\partial \rho}{\partial t} + \nabla(\rho q) = 0 \quad \text{...} 2.2$$

In this case study, it is assumed the fluid is incompressible and so mass, volume and hence density remains constant. The above equation is a partial differential equation and therefore it means that the velocity is continuous.

Since the flow under consideration is incompressible, then density is taken as a constant so that $\frac{\partial \rho}{\partial t} = 0$ and the above equation (2.2) reduces to

$$\nabla . q = 0 \quad \text{...} 2.3$$

2.1.2 Momentum equation

The principle of conservation of momentum is the application of Newton's second law of motion to an object (or element of any fluid) which is explained or stated as the rate at which momentum of the fluid is changing equal to the net external force acting on the mass.

The momentum equation can be written as

$$\frac{Du}{Dt} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 u + \vec{F} + \vec{F}_e \quad \text{...} 2.4$$

The body force $\vec{F}$ comes from gravitational force, ($\vec{F} = g\alpha\beta\nabla T$) and the electromagnetic force ($\vec{F}_e = \vec{J} X \vec{B}$) and finally the above equation can be re-written as

$$\rho\left(\frac{\partial\vec{q}}{\partial t} + \vec{q}\cdot\nabla\vec{q}\right) = \mu\nabla^2\vec{q} + g\,p\beta\left(T_s^* - T_\infty^*\right) + \vec{J}\times\vec{B} \quad \text{...2.5}$$

2.1.3 Conservation of energy equation

This equation results from the first law of thermodynamics which states that the amount of heat added to a system dQ equals to the change in internal energy de plus the work done dW i.e. $dQ = de + dW$. The energy equation can be expressed mathematically as:

$$dQ = d\,e + p\,dV \quad \text{...2.6}$$

The above equation can be re-written as

$$\rho\left(\frac{\partial h}{\partial t} + u.\nabla h\right) = Q - \nabla.q + \left(\nabla u : \sigma + \frac{\partial\rho}{\partial t} + \nabla.\rho u\right) \quad \text{...2.7}$$

and where tensor identity $\nabla u : \sigma$ is given by

$$\nabla u : \sigma = \nabla.(u.\sigma) - u(\nabla.\sigma) \quad \text{...2.8}$$

The equation above need to be simplified to contain measurable variables as Temperature. The term from heat generation is specified independently. Heat flux due to radiation is assumed to be negligible. Conductive heat flux which is formulated by Fourier's law is expressed as

$$q = -k\nabla T \quad \text{.......... 2.9}$$

Where q is conductive heat flux, k is coefficient of thermal conductivity of the fluid and T is the temperature of the fluid. Enthalpy is directly proportional to temperature and pressure and can be written as

h = h (T, P)…………………………………………………………………2.10

Specific heat capacity found at some mean pressure in the fluid flow is defined as

$C_p = C_P = \frac{\partial h}{\partial T}\Big|_p$ 2.11

The dissipative heat generated by electric current is given as $\frac{j^2}{\sigma}$.

Now, Equation 2.7 simplifies to;

$$\rho C_p = \left(\frac{\partial T}{\partial t} + u.\nabla T\right) = \nabla.K\nabla T + Q + \frac{j^2}{\sigma} \quad \text{.......... 2.12}$$

If it is assumed that Q=0 and $\frac{j^2}{\sigma} = 0$ (since it is assumed that no heat is dissipated) for a range of temperatures, then the equation of Newtonian fluid (Fluids for which the shearing stress is linearly related to the rate of shearing strain) become;

$$\frac{\partial \mathrm{T}}{\partial t} + u \cdot \nabla \mathrm{T} = \frac{K}{\rho C_p}\nabla^2 \mathrm{T} \quad \text{.......... 2.13}$$

and hence equation 2.13 becomes;

$$\frac{\partial \mathrm{T}}{\partial t} + u \cdot \nabla \mathrm{T} = \alpha \nabla^2 \mathrm{T} \quad \text{.......... 2.14}$$

Where $\alpha = \frac{k}{\rho C_p}$

2.2 ELECTROMAGNETISM EQUATIONS

Electromagnetism constitutes of interaction between rapid varying electric and magnetic fields. Using electromagnetic theory, the following equations which are named as Maxwell's equations are formulated;

$$\left.\begin{aligned} &\nabla \cdot \vec{B} = 0 \\ &\nabla \times \mathrm{E} + \frac{\partial \vec{B}}{\partial t} = 0 \\ &\nabla \cdot \vec{j} = 0 \\ &\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = J \end{aligned}\right\} \qquad \text{2.15}$$

Where $\vec{B} = \mu_o \vec{H}$ 2.16

Where $\vec{B}$ is magnetic field flux, Wb, $\vec{H}$ is magnetic field intensity, wbm^{-2}

$\vec{J} = \sigma E$ 2.17

2.3 OHM'S LAW

When magnetic field is present in an electrically conducting fluid or a time varying magnetic field is present in a stationary electrically conducting fluid, electromotive force (e.m.f) is induced and magnitude of e.m.f is directly proportional to the size of magnetic field, B and velocity of the fluid, V. The induced e.m.f is given as $\vec{V} \times \vec{B}$.

E.m.f = $\vec{V}\times\vec{B}$ ……………………………………………………………………………2.18

The current density, J of stationary electrically conducting fluid is expressed as:

$$\vec{J} = \sigma\vec{E} \quad \text{……………………………………………………………………2.19}$$

Where displacement current, D is negligible as in the MHD cases.

In general, current density J, of an electrically conducting fluid (due to an externally applied magnetic field) is expressed as

$$\vec{J} = \sigma\left(\vec{E}+\vec{V}\times\vec{B}\right) \quad \text{………………………………………………………………..2.20}$$

2.4 INDUCTION EQUATION

The generalized Ohms law is

$$J = \sigma(E + v\times B) + \rho_e v \quad \text{………………..2.21}$$

Neglecting the displacement current and taking the curl of equation yields

$$\nabla\times J = \sigma[\nabla\times E + \nabla\times(v\times B)] \quad \text{………………..2.22}$$

Where curl means the cross product of del and a vector. Substituting the Maxwell's equation (2.15) and (2.17) yields

$$\nabla\times\left(\nabla\times\vec{H}\right) = \sigma\left[-\mu\frac{\partial\vec{H}}{\partial t} + \nabla\times\left(\vec{v}\times\vec{B}\right)\right] \quad \text{………………..2.23}$$

and substituting $B = \mu_e H$ and simplify using cross-product rule

$$a\times(b\times c) = (a.c)b - (a.b)c$$

$$\frac{1}{\mu_e}\nabla\times(\nabla\times B)=\frac{1}{\mu_e}[(\nabla.B)\nabla-(\nabla.\nabla)B]=-\frac{1}{\mu_e}\nabla^2 B \quad Since\nabla.B=0$$. LHS

………………..2.24

$$-\sigma\mu_e\frac{\partial H}{\partial t}+\sigma\mu_e\nabla\times(v\times H)=-\eta\frac{\partial H}{\partial t}+\eta Curl(v\times H)$$ RHS ………………..2.25

$$\frac{1}{\mu_e}\left(-\nabla^2 B\right)=-\sigma\mu_e\frac{\partial H}{\partial t}+\sigma\mu_e\nabla\times(v\times H)$$ ………………..2.26

Substituting $B=\mu_e H$ in 2.26 yields

$$\left.\begin{aligned}\frac{\partial H}{\partial t}&=\eta Curl(v\times H)+\nabla^2 H\\ \frac{\partial H}{\partial t}&=Curl(v\times H)+\frac{1}{\eta}\nabla^2 H\end{aligned}\right\}$$ ………………..2.27

Where $\eta=\sigma\mu_e$`

Expanding the first term on RHS of equation 2.27 using vector cross product yields

$$\frac{\partial H}{\partial t}=(\nabla.H)q-(\nabla.q)H+\frac{1}{\eta}\nabla^2 H$$ ………………..2.28

The induction equation is expressed as

$$\frac{\partial H}{\partial t}=(q.\nabla)H-(H.\nabla)q=\vartheta_H\nabla^2 H$$ ………………..2.29

The constant ϑ_H is called the magnetic diffusivity and σ is the reciprocal of electrical diffusivity of the fluid. Substituting $\vartheta_H=(\rho\mu_e)^{-1}$ in the above equation yields

$$\frac{\partial H}{\partial t}=(q.\nabla)H-(H.\nabla)q=\frac{1}{\sigma\mu_e}\nabla^2 H$$ ………………..2.30

Neglecting the magnetic diffusivity (since as $\eta=\sigma\mu_e\to\infty$ as $\sigma\to\infty$ then $1/\sigma\mu_e\to 0$ and hence $0\times\Delta^2 H=0$)

The induction equation (2.29) is modified by substituting the magnetic field intensity H with the magnetic induction vector B. We consider that B is in the direction of H and $B=\mu_e H$ yielding

$$\frac{\partial B}{\partial t}-(q\cdot\nabla)B-(B\cdot\nabla)q=0 \qquad \text{………………..……….2.31}$$

Simplifying the induction equation (2.31) using the dot product rule yields

$$\frac{\partial \mu_e\vec{H}}{\partial t}+(\vec{u}\cdot\nabla)\mu_e\vec{H}-(\mu_e\vec{H}\cdot\nabla)\vec{u}=\frac{1}{\sigma\mu_e}\nabla^2\mu_e\vec{H} \qquad \text{……………………………………..2.32}$$

$$\mu_e\frac{\partial \vec{H}}{\partial t}+\mu_e(\vec{u}\cdot\nabla)\vec{H}-\mu_e(H\cdot\nabla)\vec{u}=\frac{1}{\sigma}\nabla^2\vec{H}$$

$$\mu_e\left[\frac{\partial \vec{H}}{\partial t}+(\vec{u}\cdot\nabla)\vec{H}-(\vec{H}\cdot\nabla)\vec{u}\right]=\frac{1}{\sigma}\nabla^2\vec{H}$$

$$\frac{\partial \vec{H}}{\partial t}+(\vec{u}\cdot\nabla)\vec{H}-(H\cdot\nabla)\vec{u}=\frac{1}{\sigma\mu_e}\nabla^2\vec{H} \qquad \text{………………………………………………...2.33}$$

This equation can be expanded as follows:

$$\left.\begin{aligned}
&i\frac{\partial H_x}{\partial t}+j\frac{\partial H_y}{\partial t}+k\frac{\partial H_z}{\partial t}+u\frac{\partial H_x}{\partial x}+v\frac{\partial H_x}{\partial y}+w\frac{\partial H_x}{\partial z}+u\frac{\partial H_y}{\partial x}+v\frac{\partial H_y}{\partial y}+w\frac{\partial H_y}{\partial z}+u\frac{\partial H_z}{\partial x}+v\frac{\partial H_z}{\partial y}\\
&+w\frac{\partial H_z}{\partial z}-H_x\frac{\partial u}{\partial x}-H_y\frac{\partial u}{\partial y}-H_z\frac{\partial u}{\partial z}-H_x\frac{\partial v}{\partial x}-H_y\frac{\partial v}{\partial y}-H_z\frac{\partial v}{\partial z}-H_x\frac{\partial w}{\partial x}-H_y\frac{\partial w}{\partial y}-H_z\frac{\partial w}{\partial z}\\
&=\frac{1}{\sigma\mu_e}\left[i\left(\frac{\partial^2 H_x}{\partial x^2}+\frac{\partial^2 H_x}{\partial y^2}+\frac{\partial^2 H_x}{\partial z^2}\right)+j\left(\frac{\partial^2 H_y}{\partial x^2}+\frac{\partial^2 H_y}{\partial y^2}+\frac{\partial^2 H_y}{\partial z^2}\right)+k\left(\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}+\frac{\partial^2 H_z}{\partial z^2}\right)\right]
\end{aligned}\right\} \qquad \text{………...2.34}$$

Using the fact that $H = (H_x, H_y, H_z) = (0,0,H_z)$ and that the fluid flow depends on z and t only while the applied magnetic field depends on x and t, then $u = w = 0$. Then the above equation reduces to

$$\frac{\partial B_y}{\partial t} - u\frac{\partial B_y}{\partial x} - B_y\frac{\partial u}{\partial y} = 0 \qquad \text{.....................................2.35}$$

2.5 FINAL SET OF GOVERNING EQUATIONS

The study in this thesis is MHD free convection heat and mass transfer of a heat generating fluid past a semi-infinite vertical porous plate with variable suction. A strong variable magnetic field $\vec{H}$ is applied to the semi-infinite vertical plate with variable suction. We chose the y-axis to be the co-ordinate along the plate and the x-axis to be the co-ordinate perpendicular to the plate. The plate is semi-infinite in x direction thus the x-component of the velocity profiles is invariant in the direction parallel to the plate. The equation of continuity is

$$\nabla.\vec{q} = 0 \text{...2.36}$$

The body force term is included and is given as

$$F = \rho g \text{...2.37}$$

Density variations are essential and are included

$$\rho = \rho_\circ + D\rho \text{...2.38}$$

Where $\rho_\circ$ is a constant.

The gravitational acceleration is derived from a potential $g = -\nabla\Phi$ and so

$$F = -\rho g = -(\rho_\circ + D\rho)\nabla\Phi = -\nabla(\rho.\Phi) + D\rho g \quad \ldots\ldots 2.39$$

The density, ρ is directly proportional to temperature, T

$$D\rho = \alpha\rho_\circ DT \quad \ldots\ldots 2.40$$

The momentum equation become

$$\frac{Du}{Dt} = -\frac{1}{P}\nabla \mathrm{P} + \nu\nabla^2 u + g\alpha\nabla T + \vec{J}\times\vec{B} \quad \ldots\ldots 2.41$$

Where α is the coefficient of expansion of the fluid.

The momentum, energy and induction equations governing the fluid flow are as shown below:

Momentum: $$\rho\left(\frac{\partial \vec{q}}{\partial t} + \vec{q}\nabla\vec{q}\right) = \mu\nabla^2\vec{q} + g\,p\beta\left(\mathrm{T}^* - \mathrm{T}^*_\infty\right) + \vec{J}\times\vec{B} \quad \ldots\ldots 2.42$$

Energy: $$\frac{\partial \mathrm{T}}{\partial t} + u.\cdot\nabla\mathrm{T} = \frac{K}{\rho C_p}\nabla^2\mathrm{T} \quad \ldots\ldots 2.43$$

Induction: $$\frac{\partial H}{\partial t} + \left(\vec{u}\cdot\nabla\right)\vec{H} - \left(\vec{H}\cdot\nabla\right)\vec{u} = \vartheta_H\nabla^2 H \quad \ldots\ldots 2.44$$

2.6 ANALYSIS OF THE PROBLEM

In this section, mathematical analysis and method of solution are presented. The flow of a viscous incompressible free convection heat generating fluid past an impulsively started semi-infinite vertical porous plate with variable suction in presence of variable magnetic field is considered.

Consider flow of a viscous incompressible MHD free convection heat generating fluid past an impulsively started semi-infinite vertical porous plate with variable suction in presence of variable magnetic field. The plate is suddenly set into motion in its own plane with constant velocity. It is assumed that the variable magnetic field is applied perpendicular to the direction of the flow as illustrated in the figure below (Fig 2.1)

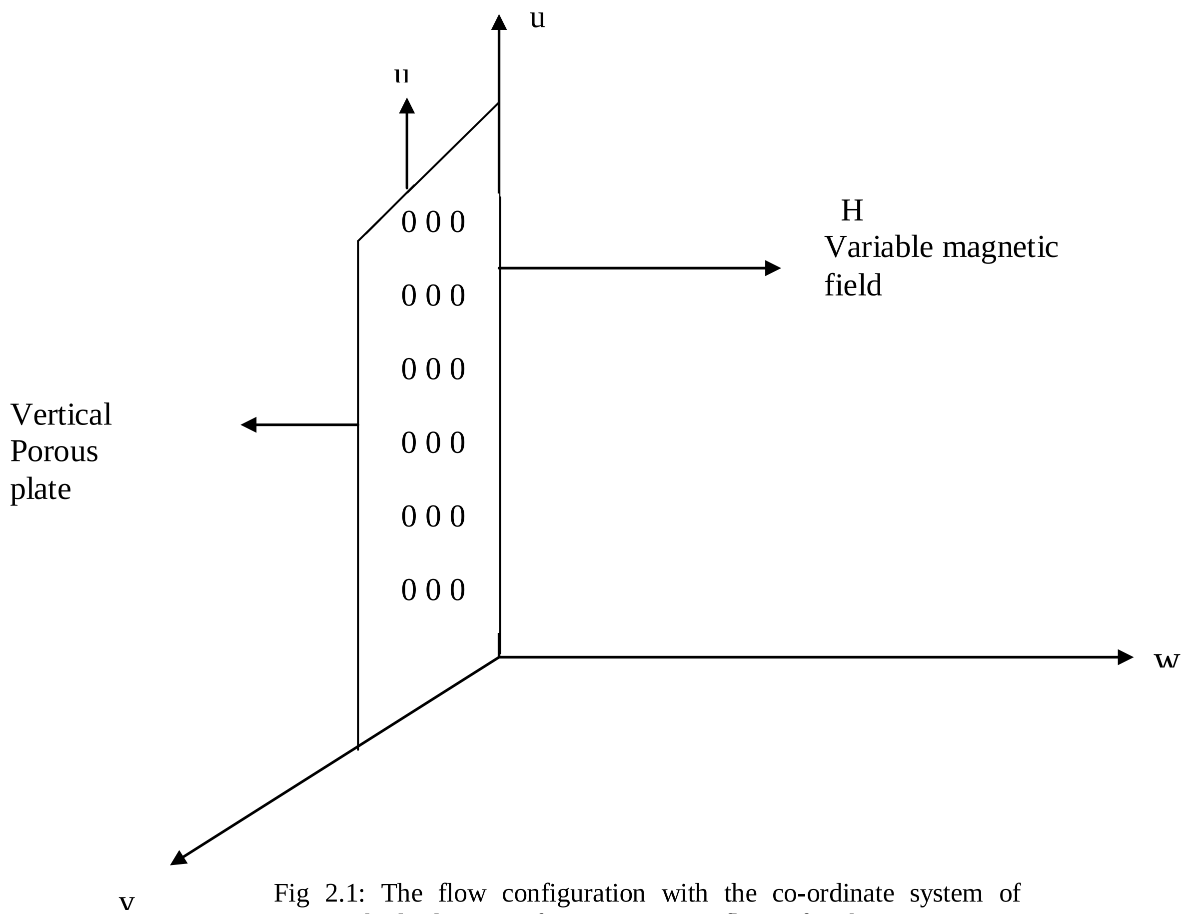


Fig 2.1: The flow configuration with the co-ordinate system of magnetohydrodynamic free convention flow of a heat generating fluid past a semi-infinite vertical porous plate with variable suction

An assumption is made that the induced magnetic field is negligible such that $H = (0,0,H_z)$. This assumption is justified owing to the fact that the Magnetic Reynold's number of a partially ionized fluid is very small.

We consider the system to be rotating with uniform angular velocity Ω about the z-axis which is taken normal to the plate. Since the plate is semi-infinite in length, the variables are functions of y^+ and t^+ only. At time $t^+ > 0$, the plate start moving impulsively in its own plane with constant velocity v_o and its temperature instantaneously increased or decreased to T^{+}_{w} which is maintained constant later. Initially the temperature of the fluid and the plate are assumed to be the same.

The equation of conservation of electric charge $\nabla.\vec{J}=0$ gives $\vec{J}z^{+}=Constant$ where $\vec{J}=\left(Jx^{+},Jy^{+},Jz^{+}\right)$. This constant is assumed to be equal to zero since $Jz^{+}=0$ at the plate which is electrically non-conducting. Thus $Jz^{+}=0$ everywhere in the flow. The generalized ohm's law must be modified to include the effects of Hall currents and variable magnetic field as follows:

$$\vec{J}+\frac{\omega_e\tau_e}{H_o}\vec{J}\times\vec{H}_y=\sigma\left[\vec{E}+\mu_e\vec{q}\times H_y+\frac{1}{e\eta_e}\nabla.P_e\right] \quad \text{2.45}$$

Where $\sigma,\mu_e,\tau_e,e,\eta_e$ and P_e is the electric conductivity, the magnetic permeability, the cyclotron frequency, the collision time, the electric charge, the number density of electron and the electron pressure respectively.

It is assumed that $\omega_e\tau_e\leq 1$. The induced magnetic field is assumed to be zero and the pressure gradient may be neglected. The ion-slip and thermoelectric effects are also neglected. Thus equation (2.45) yields

$$\left(J_x^{+},J_y^{+}\right)+\frac{\omega_e\tau_e}{\vec{H}_o}\left(J_y^{+}H_o,-J_x^{+}H_o\right)=\sigma\mu_e\left(v^{+}H_o,-u^{+}H_o\right) \quad \text{2.46}$$

Equating the x^{+} and the y^{+} component in the above equation gives:

$$\left.\begin{aligned}Jx^{+}+\omega e\,\tau eJy^{+}&=\sigma\mu ev^{+}\vec{H}o\\ Jy^{+}-\omega e\,\tau eJx^{+}&=-\sigma\mu eu^{+}\vec{H}o\end{aligned}\right\} \quad \text{2.47}$$

Solving for J_x^{+} and J_y^{+} by first eliminating J_y^{+} yields

$$Jx^{+} = \frac{\sigma o \mu e Ho}{1+m^{2}}\left(v^{+} + mu^{+}\right) \quad \text{...2.48}$$

And consequently eliminating $J_{x^{+}}$

$$J_{y}{}^{+} = \frac{\sigma_{o}\mu_{e}H_{o}}{1+m^{2}}\left(mv^{+} - u^{+}\right) \quad \text{...2.49}$$

When the effect of rotation is considered, the Coriolis force has to be included in the momentum equation. Considering a rotating frame of reference with a uniform angular velocity, Ω, the equations of motion become:

$$\frac{\partial u^{+}}{\partial t^{+}} - wo^{+}\frac{\partial u^{+}}{\partial z^{+}} - 2\Omega v^{+} = \nu\frac{\partial^{2}u^{+}}{\partial z^{+2}} + g\beta^{*}\left(\mathrm{T}^{+} - \mathrm{T}^{+}{}_{\infty}\right) + \frac{\mu_{e}J_{y^{+}}H_{o}}{\rho} + g\beta^{*}\left(C^{+} - C^{+}{}_{\infty}\right) \quad \text{.......................2.50}$$

$$\frac{\partial v^{+}}{\partial t^{+}} - w_{o}\frac{\partial v^{+}}{\partial z^{+}} + 2\Omega u^{*} = \nu\frac{\partial^{2}u^{+}}{\partial z^{+2}} - \frac{\mu_{e}J_{x^{+}}H_{o}}{\rho} \quad \text{..................................2.51}$$

$$\frac{\partial \mathrm{T}^{+}}{\partial t^{+}} - w_{o}\frac{\partial \mathrm{T}^{+}}{\partial z^{+}} = \frac{\kappa}{\rho C_{p}}\frac{\partial^{2}\mathrm{T}^{+}}{\partial z^{+2}} + \frac{\nu}{Cp}\left[\left(\frac{\partial v^{+}}{\partial z^{+}}\right)^{2} + \left(\frac{\partial u^{+}}{\partial z^{+}}\right)^{2}\right] \quad \text{..................................2.52}$$

$$\frac{\partial C^{+}}{\partial t^{+}} - w_{o}\frac{\partial C^{+}}{\partial Z^{+}} = D\frac{\partial^{2}C^{+}}{\partial z^{+2}} \quad \text{...2.53}$$

$$\frac{\partial B_{y}^{+}}{\partial t^{+}} + u^{+}\frac{\partial B_{y}^{+}}{\partial z^{+}} - B_{y}^{+}\frac{\partial u^{+}}{\partial y^{+}} = 0 \quad \text{....................................2.54}$$

In the dimensionalization process the following set of general scaling variables are utilized in our study

$$
\left.\begin{array}{l}
t=\dfrac{t^{+}U^{2}{}_{0}}{\nu},\ z=\dfrac{z^{+}U_{0}}{\nu},\ u=\dfrac{u^{+}}{U_{0}},\ w_{o}=\dfrac{wo^{+}}{uo} \\
v=\dfrac{v^{+}}{u_{0}},\ x=\dfrac{x^{+}U^{2}{}_{0}}{\nu},\ \theta=\dfrac{T^{+}-T^{+}{}_{\infty}}{T^{+}{}_{w}-T^{+}{}_{\infty}},\ y=\dfrac{y^{+}u_{o}{}^{2}}{\nu} \\
G_{r}=\nu g\beta'\dfrac{\left(T^{*}{}_{w}-T^{*}{}_{\infty}\right)}{u_{o}{}^{3}},\ \mathrm{P_{r}}=\dfrac{\mu C_{p}}{\kappa},\ Sc=\dfrac{D}{\nu} \\
M^{2}=\dfrac{\sigma H_{o}{}^{2}\mu_{e}{}^{2}\nu}{\rho u_{o}{}^{2}},\ E_{r}=\dfrac{\Omega\nu}{u_{o}{}^{2}},\ \mathrm{P_{r}}=\dfrac{\mu C_{p}}{\kappa},\ C=\dfrac{C^{+}-C^{+}{}_{\infty}}{C^{+}{}_{w}-C^{+}{}_{\infty}}
\end{array}\right\}\ \ldots\ldots\ldots\ldots..2.55
$$

The initial boundary conditions are

For $t<0$:

$$u^{+}(z^{+},t)=0,\ v^{+}(z^{+},t)=0,\ T^{+}(z^{+},t)=T^{+}{}_{\infty},\ C^{+}(z^{+},t)=C^{+}{}_{\infty}\ \ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ldots\ldots2.56$$

For $t>0$:

$$u^{+}(0,t)=U,\ v^{+}(0,t)=0,\ T^{+}(0,t)=T^{+}{}_{w},\ C^{+}(0,t)=C^{+}{}_{w}\quad\ldots\ldots\ldots\ldots\ldots\ldots...\ldots\ldots\ldots2.57$$

$$u^{+}(\infty,t)=0,\ v^{+}(\infty,t)=0,\ T^{+}(\infty,t)=T^{+}{}_{\infty},\ C^{+}(\infty,t)=C^{+}{}_{\infty}\quad\ldots\ldots\ldots\ldots\ldots\ldots...\ldots\ldots\ldots2.58$$

Let Q^{+} be the internal heat generation. The internal heat generation is assumed to be of the form

$$Q^{+}=-\left(T^{+}-T^{+}{}_{\infty}\right)Q\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots2.59$$

On substituting equations (2.48) and (2.49) into equations (2.50) and (2.51), the system of equations governing the problem becomes:

$$\frac{\partial u^{+}}{\partial t^{+}}-wo^{+}\frac{\partial u^{+}}{\partial z^{+}}-2\Omega v^{+}=g\beta^{*}\left(T^{+}-T^{+}{}_{\infty}\right)+g\beta^{*}\left(C^{+}-C^{+}{}_{\infty}\right)+\nu\frac{\partial^{2}u^{+}}{\partial z^{+2}}+\frac{\mu_{e}{}^{2}J_{y}{}^{+}H^{2}\sigma}{\rho\left(1+m^{2}\right)}\left(mv^{+}-u^{+}\right)\ldots\ldots...\ldots...2.60$$

$$\frac{\partial v^+}{\partial t^+} - wo\frac{\partial v^+}{\partial z^+} - 2\Omega v^+ = \nu\frac{\partial^2 u^+}{\partial {z^+}^2} - \frac{\mu_e J_{x^+} H_o{}^2 \sigma}{\rho\left(1+m^2\right)}\left(mu^+ + v^+\right) \quad \text{…....….2.61}$$

$$\frac{\partial T^+}{\partial t^+} - wo\frac{\partial T^+}{\partial z^+} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^+}{\partial {z^+}^2} + \frac{Q^+}{\rho C_p} \quad \text{……………………………………...…2.62}$$

$$\frac{\partial C^+}{\partial t^+} - wo\frac{\partial C^+}{\partial Z^+} = D\frac{\partial^2 C^+}{\partial {z^+}^2} \quad \text{………………………………………..…….2.63}$$

$$\frac{\partial B_y^+}{\partial t^+} + u^+\frac{\partial B_y^+}{\partial z^+} - B_y^+\frac{\partial u^+}{\partial y^+} = 0 \quad \text{…………………...……………..2.64}$$

2.7 NON- DIMENSIONAL PARAMETERS

The following non-dimensional numbers play a significant role in this study:

2.7.1 Magnetic Reynolds number

It represents the diffusion of magnetic field

$$R\sigma = \sigma\mu_e u_0 L = u_0 L = \frac{u_0 L}{\upsilon_H}$$

2.7.2 Prandtl number

This is a non-dimensional parameter which represents the ratio of momentum diffusivity ν to thermal diffusivity, κ this number is given by;

$$P_r = \frac{\nu\rho C_p}{k} = \frac{\mu C_p}{k}$$

2.7.3 Grashof number

This gives the ratio of buoyancy to viscous force;

$$G_r = \frac{vg\beta[T_S^* - T_\infty^*]}{V_0^3}$$

2.7.4 Eckert number

This is a measure of kinetic energy of the flow relative to the enthalpy differences across the field;

$$Ec = \frac{U_0^2}{C_p\left(T_S^* - T_\infty^*\right)}$$

2.7.5 Hartmann number

It is the ratio of magnetic force to viscous force;

$$\text{M}= \mu \; H \, U\sqrt{\frac{\sigma}{\mu}} = \sqrt{\frac{\sigma\mu_c^2 \, H^2 V}{\left(\frac{\mu}{\nu}\right)U_0^2}} = \sqrt{\frac{magnetic \;\; force}{Viscous \; force}}$$

2.7.6 Magnetic parameter

It is the ratio of magnetic force to inertial force

$$M_1 = \sqrt{\frac{\sigma\mu_e^2 H^2 \nu}{\frac{\mu U_0^2}{\nu}}} = \sqrt{\frac{magnetic \; force}{inertia \;\; force}}$$

2.7.7 Schmidt number, Sc

This is the ratio of relative velocity and contraction boundary layer thickness for convection mass transfer in laminar flows.

$$S_c = \frac{\nu}{D}$$

2.7.8 Nusselt number, Nu

It provides a measure of convection heat transfer occurring at the surface;

$$N_u = \frac{hL}{k}$$

2.7.9 Rotation Parameter, Er

This gives the gross measure of how the Coriolis force compares to the viscous force. It is given by the relation;

$$E_r = \frac{\Omega v}{U_o^3}$$

2.8 THE PROCESS OF NON-DIMENSIONALIZATION SCHEME

The non-dimensionalization process allows us to apply results obtained for a surface experiencing a set of conditions to a geometrically similar surface experiencing entirely same conditions. Due to the fluid nature, size of the surface or the fluid velocity, the conditions may vary.

Consider the components of equations (2.50) and (2.51) which yield:

$$\frac{\partial u^+}{\partial t^+} = \frac{U_o}{\nu / U_o{}^2} \frac{\partial u}{\partial t} = \frac{U_o{}^3}{\nu} \frac{\partial u}{\partial t} \quad \text{.....2.65}$$

$$\frac{\partial v^+}{\partial t^+} = \frac{U_o}{\nu / U_o{}^2} \frac{\partial v}{\partial t} = \frac{U_o{}^3}{\nu} \frac{\partial v}{\partial t} \quad \text{.....2.66}$$

$$\frac{\partial^2 u^+}{\partial t^{+2}} = \frac{U_o}{\nu / U_o{}^2} \frac{\partial u}{\partial t} = \frac{U_o{}^3}{\nu} \frac{\partial^2 u}{\partial t^2} \quad \text{.....2.67}$$

$$\frac{\partial^2 u^+}{\partial z^{+2}} = \frac{\partial}{\partial z^+}\left(\frac{\partial u^+}{\partial z^+}\right) = \frac{\partial}{\partial z^+}\left(\frac{U_o{}^2}{\nu} \cdot \frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{U_o{}^2}{\nu} \cdot \frac{\partial u}{\partial z}\right)\frac{\partial z}{\partial z^+} = \frac{U_o{}^3}{\nu^2} \cdot \frac{\partial^2 u}{\partial z^2} \quad \text{.....2.68}$$

Similarly

$$\frac{\partial^2 v^+}{\partial z^{+2}} = \frac{\partial}{\partial z^+}\left(\frac{\partial v^+}{\partial z^+}\right) = \frac{\partial}{\partial z^+}\left(\frac{Uo^2}{\nu}\cdot\frac{\partial v}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{Uo^2}{\nu}\cdot\frac{\partial v}{\partial z}\right)\frac{\partial z}{\partial z^+} = \frac{Uo^3}{\nu^2}\cdot\frac{\partial^2 v}{\partial z^2} \qquad \text{..................2.69}$$

On substitution of equations2.65 to 2.69 into equations (2.50) and (2.51) we obtain

$$\frac{Uo^3}{\nu}\frac{\partial u}{\partial t} - Uowo\frac{U0^2}{\nu^2}\frac{\partial u}{\partial z} - 2\Omega uov = \nu\frac{Uo^3}{\nu^2}\frac{\partial^2 u}{\partial z^2} + g\beta^*\left(T^+ - T^+_\infty\right) + \frac{\mu eJy^+ Ho}{\rho} + g\beta^*\left(C^+ - C^+_\infty\right) + \frac{\mu eH}{\rho}jy^+ \qquad \text{.....2.70}$$

$$\frac{Uo^3}{\nu}\frac{\partial v}{\partial t} - uouwo\frac{Uo^3}{\nu}\frac{\partial v}{\partial z} - 2\Omega uuou = \frac{Uo^3}{\nu}\frac{\partial^2 u}{\partial z^2} - \frac{\mu eJx^+ H}{\rho} \qquad \text{.................2.71}$$

Dividing the equations by Uo/ν yields

$$\frac{\partial u}{\partial t} - wo\frac{\partial u}{\partial z} - \frac{2\Omega\nu\vartheta}{Uo^2} = \frac{\partial^2 u}{\partial z^2} + \frac{g\nu\beta\left(T^+ - T^+_\infty\right)}{Uo^3} + \frac{g\nu\beta\left(C^+ - C^+_\infty\right)}{Uo^3} + \frac{\mu e^2 Jy^+ H^2 \nu}{Uo^3\rho\left(1+m^2\right)}\left(mv - u\right) \qquad \text{.................2.72}$$

$$\frac{\partial v}{\partial t} - wo\frac{\partial v}{\partial z} - \frac{2\Omega\nu u}{Uo^2} = \frac{\partial^2 u}{\partial z^2} - \frac{\mu eJx^+ Ho^2 \nu}{Uo^3\rho\left(1+m^2\right)}\left(v + mu\right) \qquad \text{...........................2.73}$$

On using the non-dimensional variables defined in (2.55), equations (2.56) and (2.57) yields

$$\frac{\partial u}{\partial t} - wo\frac{\partial u}{\partial z} - 2vEr = \frac{\partial^2 u}{\partial z^2} + Gr\theta + GcC + \frac{M1^2}{\left(1+m^2\right)}\left(mv - u\right) \qquad \text{.............................2.74}$$

$$\frac{\partial v}{\partial t} - wo\frac{\partial v}{\partial z} - 2uEr = \frac{\partial^2 u}{\partial z^2} - \frac{M1^2}{\left(1+m^2\right)}\left(v + mu\right) \qquad \text{.......................................2.75}$$

On multiplication of equation (2.74) by the complex number i (the complex number defined by $i = \sqrt{-1}$) and adding to equation (2.75) we obtain:

$$\frac{\partial}{\partial t}(u+iv)-(v+ui)=\frac{\partial^2}{\partial z^2}(u+iv)+\frac{M_1^2}{1+m^2}(-u-iv+m(v-iu))+Gr+GcC \quad \ldots\ldots\ldots 2.76$$

This can be written as

$$\frac{\partial \vec{q}}{\partial t}-wo\frac{\partial \vec{q}}{\partial z}=\frac{\partial^2 \vec{q}}{\partial z^2}+Gr\theta+GcC-M^2\vec{q} \quad \ldots\ldots\ldots 2.77$$

On substitution of relevant non-dimensional variables in equation (2.62), the following is obtained;

$$\frac{\partial T^+}{\partial t^+}=\frac{\left(T^+-T^+\infty\right)}{\nu}\frac{\partial \theta}{\partial t}=\frac{Uo^2\left(T^+-T^+\infty\right)}{\nu}\frac{\partial \theta}{\partial t} \quad \ldots\ldots\ldots 2.78$$

$$\frac{\partial^2 T^+}{\partial t^{+2}}=\frac{Uo^2\left(T^+-T^+\infty\right)}{\nu}\frac{\partial^2 \theta}{\partial t^2} \quad \ldots\ldots\ldots 2.79$$

$$\left(\frac{\partial u^+}{\partial z^+}\right)^2=\frac{Uo^4}{\nu^2}\left(\frac{\partial u}{\partial z}\right)^2 \quad \ldots\ldots\ldots 2.80$$

$$\left(\frac{\partial v^+}{\partial z^+}\right)^2=\frac{Uo^4}{\nu^2}\left(\frac{\partial v}{\partial z}\right)^2 \quad \ldots\ldots\ldots 2.81$$

And

$$\frac{\partial T^+}{\partial z^+}=\frac{Uo\left(T^+-T^+\infty\right)}{\nu}\frac{\partial \theta}{\partial z} \quad \ldots\ldots\ldots 2.82$$

$$\frac{Uo^2\left(T^+ - T^+{}_\infty\right)}{\nu}\frac{\partial\theta}{\partial t} - \frac{UowoUo\left(T^+ - T^+{}_\infty\right)}{\nu}\frac{\partial\theta}{\partial z} = \frac{\kappa}{\rho Cp}Uo^2\frac{\left(T^+ - T^+{}_\infty\right)}{\nu}\frac{\partial^2\theta}{\partial t^2} + \frac{\nu}{Cp}\left[\frac{Uo^4}{\nu^2}\left(\frac{\partial v}{\partial z}\right)^2 + \frac{Uo^4}{\nu^2}\left(\frac{\partial u}{\partial z}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.83$$

By multiplying equation (2.83) by $\dfrac{\nu}{Uo^2\left(T^+ - T^+{}_\infty\right)}$ yields

$$\frac{\partial\theta}{\partial t} - wo\frac{\partial\theta}{\partial z} = \frac{\kappa}{\rho\nu Cp}\frac{\partial^2\theta}{\partial t^2} + \frac{Uo^2}{Cp\left(T^+ - T^+{}_\infty\right)}\left[\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.84$$

On using the defined non-dimensional parameters defined, equation (2.84) yields:

$$\frac{\partial\theta}{\partial t} - wo\frac{\partial\theta}{\partial z} = \frac{1}{\Pr}\frac{\partial^2\theta}{\partial t^2} + Ec\left[\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.85$$

This simplifies to

$$\Pr\frac{\partial\theta}{\partial t} - \Pr wo\frac{\partial\theta}{\partial z} = \frac{\partial^2\theta}{\partial t^2} + \Pr Ec\left[\left(\frac{\partial\vec{q}}{\partial z}\right)\left(\frac{\partial q}{\partial z}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.86$$

Similarly from equation (2.53) we have

$$C^+ - C^+{}_\infty = C\left(Cw^+ - C^+\infty\right)$$

$$C^+ = C\left(Cw^+ - C^+\infty\right) + C_w^+$$

and $t^+ = \dfrac{tv}{uo^2}$, $\quad Z^+ = \dfrac{Zv}{uo}$, Hence

$$\frac{\partial C^+}{\partial t^+} = \frac{\partial C}{\partial t}\cdot\frac{\partial C^+}{\partial C}\cdot\frac{\partial t^+}{\partial t} = \frac{u_\circ^2}{v}\left(C_w^+ - C_\infty^+\right)\frac{\partial C}{\partial t}$$

$$\frac{\partial C^+}{\partial z^+} = \frac{\partial C}{\partial z^+}\cdot\frac{\partial C^+}{\partial C}\cdot\frac{\partial z^+}{\partial z} = \frac{u_\circ^2}{v}\left(C_w^+ - C_\infty^+\right)\frac{\partial C}{\partial z}$$

$$\frac{\partial^2 C^+}{\partial {z^+}^2} = \frac{\partial}{\partial z^+}\left(\frac{\partial C^+}{\partial z^+}\right) = \frac{\partial}{\partial z^+}\left(\frac{u_\circ}{v}\left(C_w^+ - C_\infty^+\right)\right)$$

$$= \frac{\partial}{\partial z}\left(\frac{u_\circ}{v}\left(C_w^+ - C_\infty^+\right)\right)\frac{\partial z}{\partial z^+} = \frac{u_\circ}{v}\left(C_w^+ - C_\infty^+\right)\cdot\frac{u_\circ}{v}$$

$$= \frac{u_\circ^2}{v}\left(C_w^+ - C_\infty^+\right)\cdot\frac{\partial^2 C}{\partial z^2}$$

Substituting all these in equation (2.83) we obtain

$$\frac{u_\circ^2}{v}\left(C_w^+ - C_\infty^+\right)\frac{\partial C}{\partial t} - u_\circ w_\circ \frac{u_\circ}{v}\left(C_w^+ - C_\infty^+\right)\frac{\partial C}{\partial z} = D\frac{u_\circ^2}{v}\left(C_w^+ - C_\infty^+\right)\frac{\partial^2 C}{\partial z^2} \quad \text{………….. ……….2.87}$$

Multiplying through by $\dfrac{v}{U_o{}^2\left(T^+ - T^+{}_\infty\right)}$ yields

$$\left.\begin{aligned} \frac{\partial C}{\partial t} - w_\circ \frac{\partial C}{\partial z} &= D\frac{\partial^2 C}{\partial z^2} \\ \frac{\partial C}{\partial t} - w_\circ \frac{\partial C}{\partial z} &= Sc\frac{\partial^2 C}{\partial z^2} \end{aligned}\right\} \quad \text{…………………………..……………..2.88}$$

Where in this case $\vec{q} = u + vi$ is the complex velocity.

Again, non-dimensionalising equation (2.35) and substituting the magnetic Reynolds number yields

$$\frac{\partial B_y^+}{\partial t^+} + u^+\frac{\partial B_y^+}{\partial z^+} - B_y^+\frac{\partial u^+}{\partial y^+} = 0 \quad \text{…………………………………..………………2.89}$$

The non-dimensional forms of the initial and boundary conditions are:

$t < 0$: $q(z,0) = 0$, $\theta(z,0) = 0$, $C(z, 0) = 0$ ……………..………….2.90

$t > 0$: $q(0,t) = 1$, $\theta(0,t) = 1$, $C(0, t) = 1$ ………………...............2.91

$q(\infty,t) = 0$, $\theta(\infty,t) = 0$, $C(\infty, t) = 0$ ………………...............2.92

CHAPTER THREE

METHOD OF SOLUTION

3.0 INTRODUCTION

To effectively combine hydromagnetic theory and physical conditions of the flow, a mathematical model was formulated. Due to the presence of variable magnetic field and suction, equations that are non-linear in nature are generated that cannot be solved by analytical methods. Numerical methods such as finite difference method are used in solving these equations. The Non-linear equations are Non-dimensionalized, then the equations are written in finite difference form where they are solved iteratively using a computer program.

Finite difference method is applied to solve the system of non-linear partial differential equations governing the flow problem. Initial and boundary conditions are determined before the process of solving numerically the non-linear partial differential equations. The velocity, concentration and temperature profiles are used to solve skin friction, rate of heat transfer and mass transfer respectively. Before the system of equations is solved, the initial and boundary conditions need to be determined.

3.1 ASSUMPTIONS

The following assumptions are taken into account in order to simplify the governing equations;

a) $B = \mu_e H$, where B is magnetic field and H is magnetic field intensity.

b) The fluid is incompressible

c) The fluid flow is laminar

d) The displacement current is negligible since magnetic and electrical fields frequencies are insignificant compared to frequency of electron

e) Lorentz force $\vec{J} X \vec{B}$ due to magnetic field is dominant as compared to force due to electric field (F= $\rho_e E$ force per unit volume)

f) Boussinesq approximations for this study are:

i. The density gradients caused by temperature and concentration gradients causes buoyancy

ii. Density is directly proportional to both Temperature and concentration.

iii. Variation of density of fluid from ρ_ω is insignificant.

iv. Heat transfer by conduction and radiation is negligible.

3.2 FINITE DIFFERENCE APPROXIMATION

We consider the explicit finite difference method. In this method, partial differential equations are approximated by a set of linear equations relating to the values of the function at each mesh point and then the set of algebraic equations solved. An iteration procedure is developed that takes into account the non-linear character of the equation. We partition the interval [a, ∞] and [c, ∞] of the z and t axes respectively, into equal parts of width h and k. A grid is then defined by drawing vertical and horizontal lines through the points with co-ordinates ($z_{i,}$ t_j) such that

$$\left.\begin{array}{ll} z_i = a + ih, & i = 1,2,3......... \\ t_j = a + jk & j = 1,2,3..... \end{array}\right\} \quad ....3.1$$

with grid lines t_i and z_j intersecting at mesh points ($z_{i,}$ t_j). Using Taylor's expansion in the variables z and t at the points ($z_{i+1,}$ t_j) and ($z_{i-1,}$ t_j) about ($z_{i,}$ t_j) we obtain

$$\left.\begin{array}{l} u(i+1, j) = u(i, j) + u_z(i, j)\Delta z + \frac{1}{2} u_{zz}(i, j)(\Delta z)^2 + ... \\ u(i-1, j) = u(i, j) - u_z(i, j)\Delta z + \frac{1}{2} u_{zz}(i, j)(\Delta z)^2 + ... \end{array}\right\} \quad ....3.2$$

And

$$\left.\begin{array}{l} u(i+1, j) = u(i, j) + u_t(i, j)\Delta t + \frac{1}{2} u_{tt}(i, j)(\Delta t)^2 + ... \\ u(i-1, j) = u(i, j) - u_t(i, j)\Delta t + \frac{1}{2} u_{tt}(i, j)(\Delta t)^2 + ... \end{array}\right\} \quad ....3.3$$

In which case

$$u(i, j) = u(z_i, t_i)$$

And

$$u_z(i, j) = \frac{\partial u(z_i, t_i)}{\partial z}$$

On eliminating u_z from equation (3.2) we obtain the difference formula as:

$$\frac{\partial u(i, j)}{\partial z} = \frac{u(i+1, j) - u(i-1, j)}{2h} + O(h^2) \quad ....3.4$$

On eliminating u_z from equation (3.2) yields

$$\frac{\partial^2 u(i,j)}{\partial z^2} = \frac{u(i+1,j) - 2u(i,j) + u(i-1,j)}{h^2} + O(h^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

On eliminating u_t from equation (3.3) yields

$$\frac{\partial u(i,j)}{\partial t} = \frac{u(i+1,j) - u(i-1,j)}{2k} + O(k^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

And eliminating u_t from equation (3.3) yields

$$\frac{\partial^2 u(i,j)}{\partial t^2} = \frac{u(i+1,j) - 2u(i,j) + u(i-1,j)}{k^2} + O(k^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

yielding the first and second order central differences in z and t. By choosing small values of h and k minimizes truncation errors O (h^2) and O (k^2).

3.3 DEFINITION OF MESH

In order to give an explicit relation between the partial derivatives in the final set of equations and the function values at the adjacent nodal points, a uniform mesh is considered whereby the rectangular region of interest is subdivided into uniform rectangular elements centered about the mesh points (i, j) as depicted in Figure 3.1 below:

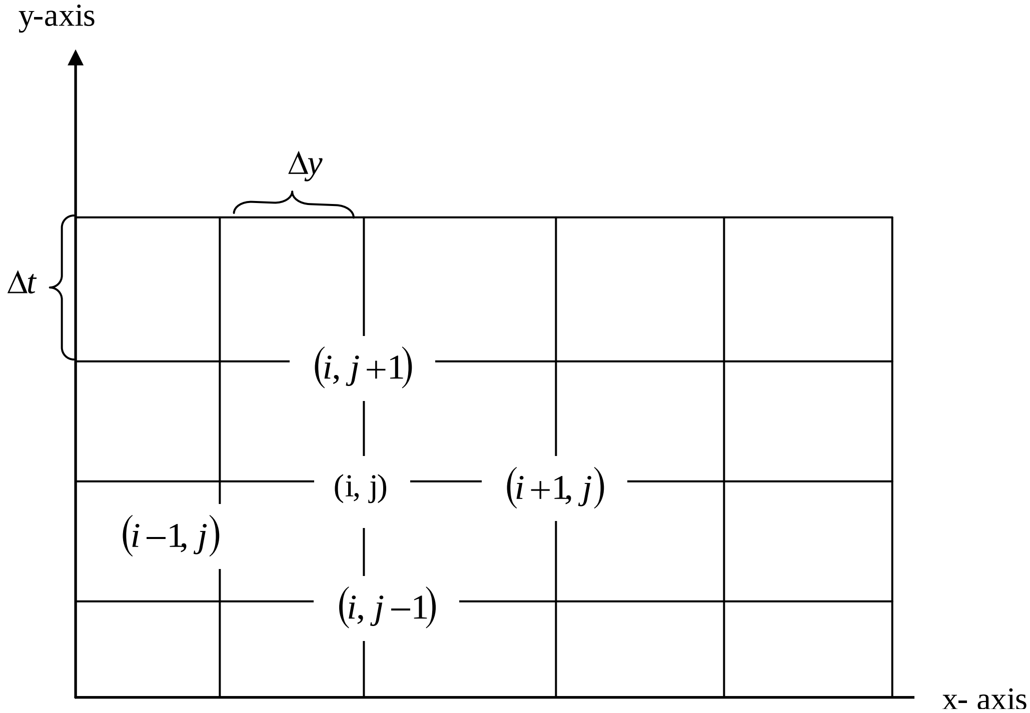


Fig 3.1 Definition of mesh

From Figure 3.1 above, the x-axis denotes variable (velocity) along the y-axis while the y-axis represents time t. The exact solutions for the equations governing our flow problem are not possible since they are non-linear. Hence to solve these equations, we use the finite difference scheme and we use the mesh system defined in Fig 3.1, the final set of non-dimensional equations which are expressed in the first and second forward difference approximation is as follows;

$$\frac{q(i, j+1)-q(i, j)}{\Delta t}=\frac{q(i+1, j)-2q(i, j)+q(i-1, j)}{(\Delta x)^2}-m^2 q+Gr\theta(i, j)+wo\frac{[q(i, j)-q(i-1, j)]}{\Delta z}$$
$$+G_C C(i, j)+\frac{\nu}{\rho uo^2}(J\times B)\ldots\ldots 3.8$$

This can be written as

$$q(i, j+1) = \Delta t\left\{ \left[\frac{q(i+1, j) - 2q(i, j) + q(i-1, j)}{(\Delta x)^2} \right] - m^2 q + Gr\theta(i, j) + w_\circ \frac{[q(i, j) - q(i-1, j)]}{\Delta z} + G_c C(i, j) + \frac{\nu}{\rho uo^2}(J \times B) \right\} + q(i, j) \quad \ldots 3.9$$

The energy equation can be expressed in the first and second difference approximation

$$\frac{\Pr\theta(i, j+1) - \theta(i, j)}{\Delta t} = \Pr w_\circ \frac{[\theta(i, j) - \theta(i-1, j)]}{\Delta z} + \frac{\theta(i+1, j) - 2\theta(i, j) + \theta(i-1, j)}{(\Delta z)^2} + \Pr Ec\left[\left(\frac{q(i+1, j) - q(i, j)}{\Delta z} \right)\left(\frac{\bar{q}(i+1, j) - \bar{q}(i, j)}{\Delta z} \right) \right] \quad \ldots 3.10$$

This can be written as

$$\theta(i, j+1) = \left\{ \begin{array}{l} \frac{\Delta t}{\Pr}\left[\frac{\Pr w_\circ [\theta(i, j) - \theta(i-1, j)]}{\Delta z} \right] + \Pr Ec\left[\left(\frac{q(i+1, j) - q(i, j)}{\Delta z} \right)\left(\frac{\bar{q}(i+1, j) - \bar{q}(i, j)}{\Delta z} \right) \right] \\ + \left[\frac{\theta(i+1, j) - 2\theta(i-1, j)}{(\Delta z)^2} \right] \end{array} \right\} + \theta(i, j)$$

……….……………………………….. 3.11

Similarly, Concentration equation can be expressed in the first and second difference approximation as;

$$\frac{C(i, j+1) - C(i, j)}{\Delta t} = \frac{w_\circ [C(i+1) - C(i-1, j)]}{\Delta z} + Sc\frac{[C(i-1, j) - 2C(i, j) + C(i+1, j)]}{(\Delta z)^2} + \frac{\nu}{\rho uo^2}(J \times B) \quad \ldots\ldots\ldots 3.12$$

This can be written as:

$$C(i, j+1) = \Delta t\left\{ \left[\frac{w_\circ [C(i, j) - C(i-1, j)]}{\Delta z} \right] + Sc\frac{[C(i-1, j) - 2C(i, j) + C(i+1, j)]}{(\Delta z)^2} + \frac{\nu}{\rho uo^2}(J \times B) \right\} + C(i, j) \quad \ldots 3.13$$

Where index i refers to z and j to time t.

Finally, the finite difference form of the governing equations is as follows;

$$\frac{q(i,j+1)-q(i,j)}{\Delta t}=w_{\circ}\frac{[q(i,j)-q(i-1,j)]}{\Delta z}+Gr\theta(i,j)+GcC(i,j)+m^2q(i,j)+\frac{[q(i-1,j)-2q(i,j)+q(i+1,j)]}{(\Delta z)^2}\text{...3.14}$$

$$\Pr\frac{\theta(i,j)-\theta(i,j)}{\Delta t}=\frac{w_{\circ}[\theta(i,j)-\theta(i-1,j)]}{\Delta z}+\frac{[\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)]}{(\Delta z)^2}+\Pr Ec\left(\frac{q(i+1,j)-q(i,j)}{\Delta z}\right)\left(\frac{\bar{q}(i+1,j)-\bar{q}(i,j)}{\Delta z}\right)\text{....3.15}$$

$$\frac{C(i,j+1)-C(i,j)}{\Delta t}=\frac{w_{\circ}[C(i,j)-C(i-1,j)]}{\Delta z}+Sc\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta z)^2}\right]\text{..........3.16}$$

$$\left[\frac{H^{i+1}(i,j)-H_y^{i}(i,j)}{\Delta t}+u^{i+1}\frac{H^{i}(i+1,j)-H^{i}(i-1,j)}{2\Delta x}\right]R_m=\frac{H^{i+1}(i+1,j)-2H^{i+1}(i,j)+H^{i+1}(i-1,j)}{(\Delta x)^2}\text{........................3.17}$$

The initial conditions at t=0 take the form

q (z, t) = 0   θ (z, t) = 0

The initial conditions at t>0 take the form

q (0, t) =1   θ (0, t) =1

The boundary conditions as z→∞ take the form

q (∞, t) =0   θ (∞, t) =0

Using these initial and boundary conditions we can compute consecutive terms of temperature, concentration and velocity profiles computed using the following finite difference form of equations;

$$q(i,j+1)=\Delta t\left\{\left[\frac{q(i+1,j)-2q(i,j)+q(i-1,j)}{(\Delta z)^2}\right]+w_{\circ}\left[\frac{q(i,j)-q(i-1,j)}{\Delta z}\right]+Gr\theta+GcC(i,j)-m_2q(i,j)\right\}+q(i,j)\text{.....3.18}$$

$$\theta(i,j+1)=\frac{\Delta t}{\Pr}\left\{\left[\frac{\Pr w_{\circ}[\theta(i,j)-\theta(i-1,j)]}{\Delta z}\right]+\left[\frac{\theta(i+1,j)-2\theta(i,j)+\theta(i-1,j)}{(\Delta z)^2}\right]+\Pr Ec\left[\left(\frac{q(i+1,j)-q(i,j)}{\Delta z}\right)\left(\frac{\bar{q}(i+1,j)-q(i,j)}{\Delta z}\right)\right]\right\}\text{.......3.19}$$

and

$$C(i, j+1) = \Delta t\left\{\left[\frac{w_{\circ}[C(i, j) - C(i-1, j)]}{\Delta z}\right] + Sc\left[\frac{C(i-1, j) - 2C(i, j) + C(i+1, j)}{(\Delta z)^2}\right]\right\} \ldots\ldots\ldots 3.20$$

$$\left[\frac{H^{i+1}(i, j) - H_y^{i}(i, j)}{\Delta t} + u^{i+1}\frac{H^{i}(i+1, j) - H^{i}(i-1, j)}{2\Delta x}\right]Rm = \frac{H^{i+1}(i+1, j) - 2H^{i+1}(i, j) + H^{i+1}(i-1, j)}{(\Delta x)^2} \ldots\ldots 3.21$$

To ensure stability and convergence of finite difference scheme, the computations are made using small values Δt. Using smaller values of Δt and consecutively increasing the number of the mesh points does not have a significant effect on the results and thus they ensure the stability and convergence of the finite difference scheme used in the present analysis.

3.4 RESULTS AND DISCUSSION

A program was written and run for different values of non-dimensional parameters to determine velocity profiles, temperature profiles and concentration profiles when the plate was conventionally heated or conventionally cooled. The Non-dimensional parameters which were used are rotational parameter, Hall parameter, Eckert number, suction parameter, magnetic parameter and injection parameter.

The velocities were classified as primary and secondary velocity along x and y-axes. Numerical computations for the velocities (both primary and secondary) profiles, temperature profiles and concentration profiles were obtained and the unsteady flow results obtained were presented in form of graphs as in Figures 4.1 to 4.8. Prandtl number Pr and magnetic parameter M used were 0.71 and 5.0 respectively. The magnetic parameter M^2 =5.0 signifies strong Magnetic field. Gr>0 (=5) corresponds to cooling of the plate by free convection currents since the plate is at a higher temperature than the surrounding and Gr<0(= -5) to heating of the plate by free convection currents since the plate is at lower temperature than the surrounding.

Case 1

In the presence of heating of the plate by free convection current for the case of Gr<0 (=-5).

From figure 4.1 and 4.2 we observe that:

- For Ec= 0.02, a decrease in the rotational parameter leads to an increase in both the primary and secondary velocity profilers. Since the wall moves in opposite direction to that of the free stream, it tends to retard the flow. Similarly, the convectional currents due to rotation cause the fluid to retard in motion.
- An increase in the Magnetic parameter leads to an increase in both the primary velocity and the secondary velocity profiles. Inclusion of Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity $\frac{\sigma}{1+m^2}$.
- We also observe that an increase in the suction parameter causes no effect in the primary velocity profile but decreases secondary velocity profiles. Introduction of suction retards fluid flow due to increase of convection currents of the fluid across the plates.

- Removal of injection increases both the primary and secondary velocity profiles. This is because the convectional currents of the fluid are reduced.
- An increase in Eckert number results to an increase in the primary velocity and secondary velocity profiles. This is due to the fact that an increase in Ec increases convectional currents which cause a slight decrease in the primary velocity.

Primary Velocity Profiles For Gr = - 5

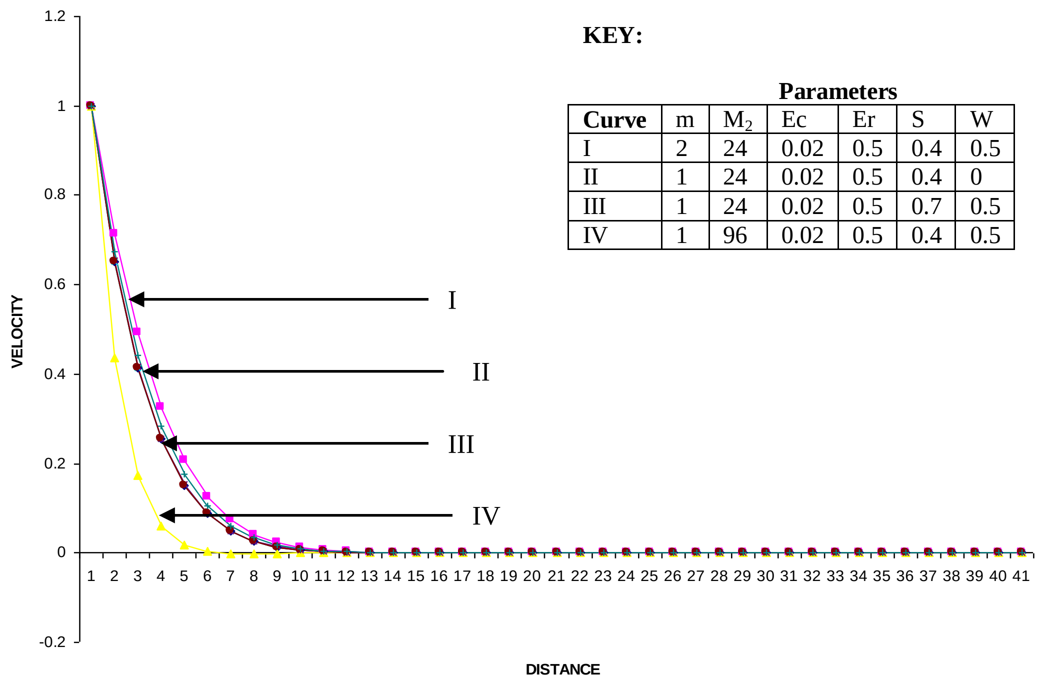


Fig 4.1: Primary velocity profiles in the presence of heating of the plate by free convection current for the case of Gr<0 (= - 5)

Secondary Velocity profiles for Gr = - 5

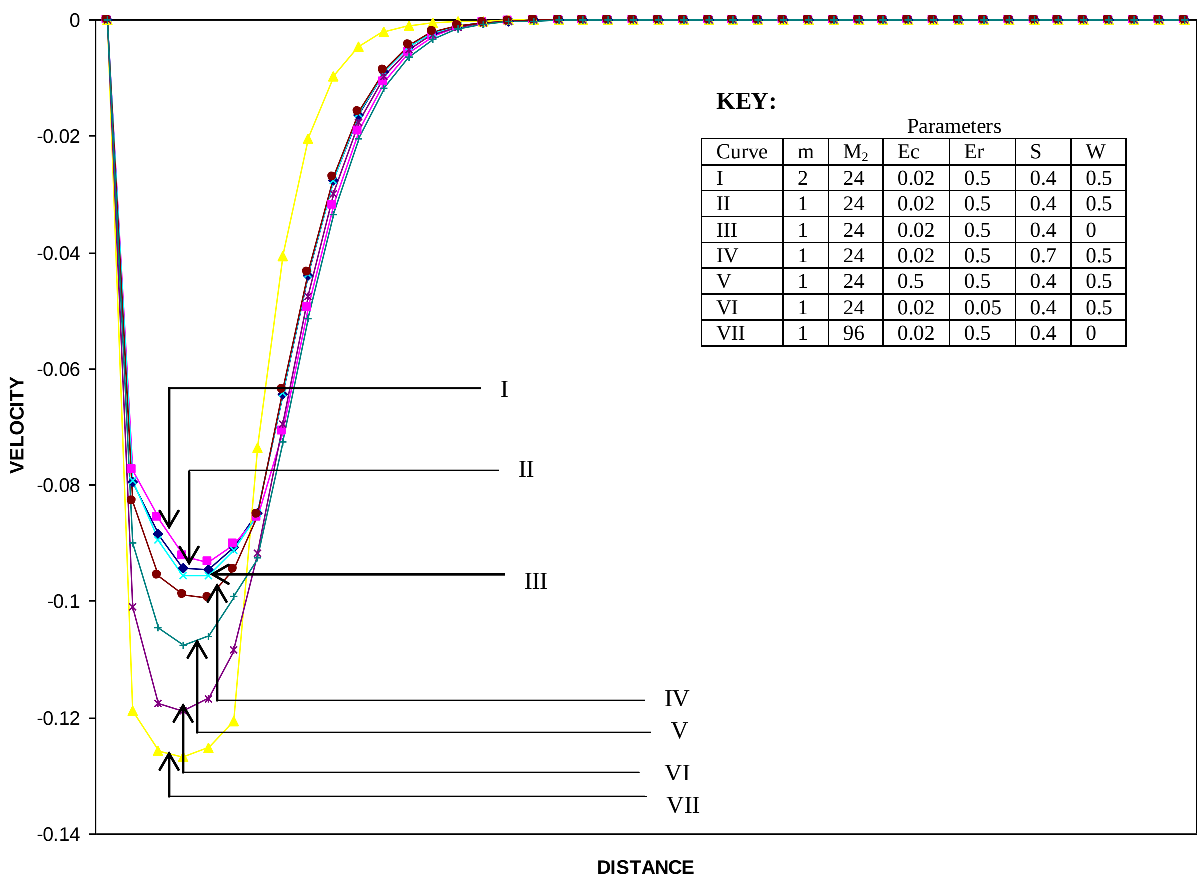


Fig 4.2: secondary velocity profiles in the presence of heating of the plate by free convection current for the case of Gr<0 (= -5)

From Fig 4.3: (Temperature profile for Gr< 0) we observe that;

- A decrease in the Rotational parameter Er gives rise to an increase in the temperature profile. The rotation causes the circulation of induced currents at the surface of the fluid, that is, the increase of the temperature affects the current distribution. Rotation leads up to additional transport; this contribution is a consequence of the decrease of the ion rotation. Viscous dissipation would immediately lead to an increase of ion-temperature, increasing ion momentum and thermal transport.
- An increase in Hall parameter causes a decrease in the temperature profile. As the distance from the plate increases, these profiles increase. However, as the distance from the plate increases these profiles remain constant. Further, an increase of Hall parameter increases cyclotron frequency and hence the rotation and collision of electrons increases. An increase in Hall parameter leads to a decrease in the effective conductivity ($\frac{\sigma}{1+m^2}$) which reduces magnetic damping force on the velocity and thus the velocity increases.
- Increase in Eckert number leads to an increase in the temperature profiles. Increasing the Eckert number causes the fluid to become warmer and therefore increase its temperature. This is attributed to the viscous dissipation.
- Increase in the Magnetic parameter leads to an increase in the temperature profiles. The increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to increase. The magnetic field

produces a huge increment in the magnitude of the temperature. This can be explained physically as follows: it is well known that a magnetic field imparts some rigidity to the conducting fluid. Thus, with increase in the magnetic field, greater effort will be necessary to maintain the rotation of the plate and this implies an increase in temperature with an increase of the parameter M.

Temperature Profiles for Gr = - 5

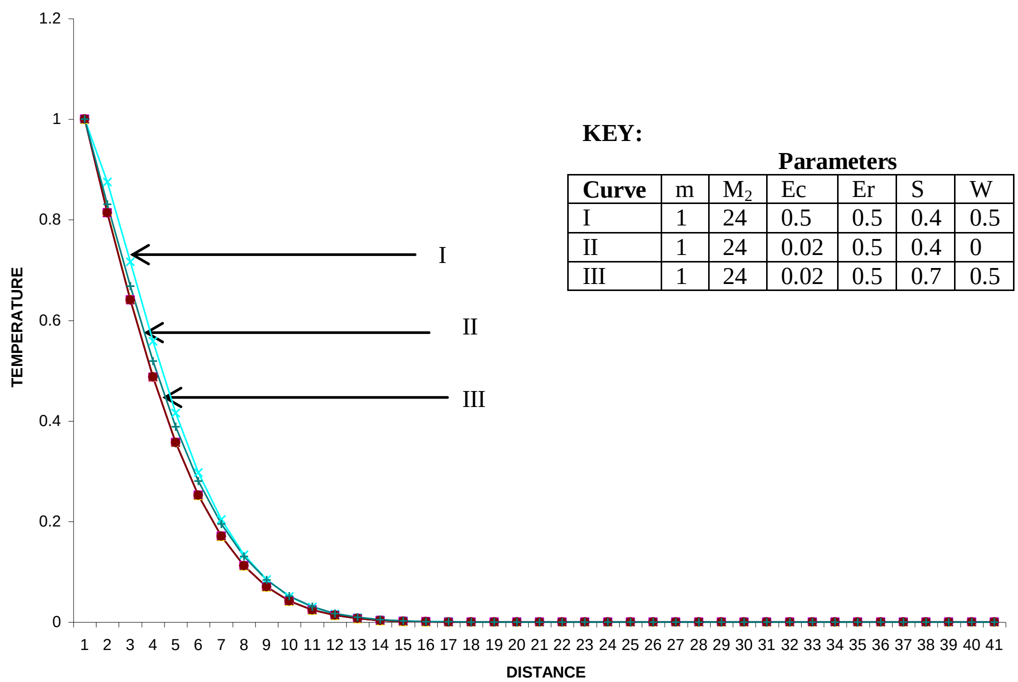


Fig 4.3: Temperature profiles in the presence of heating of the plate by free convection current for the case of Gr <0 (= -5)

From Fig 4.4: (Concentration profile for $Gr < 0$) we observe that:

- A decrease in the Rotational parameter Er has no effect to the concentration profiles. Rotation has been achieved by a transfer of angular momentum. Once this is drastically reduced, the rate at which the particles move and collide is too small such that the change is insignificant.
- An increase in Hall parameter has no effect to the concentration profiles. An increase in Hall parameter which is due to the increase of collisions has no effect to the concentration profile. This is because there is no change in the charge carriers hence the effect is neutralized. Since no polarization voltage is imposed on the fluid, the concentration profile is not affected.
- Removal of injection causes a rise in concentration profile. Removal of injection means an increase in the molecular diffusivity which consequently results in the rise of the concentration.
- Increase in Eckert number has no effect to the concentration profiles. The increase in an Eckert number increases thermal energy which consequently increases temperature and this does not affect the concentration of the fluid but increases the mass diffusion.
- Increase in suction parameter leads to a decrease in the concentration profiles. Suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth. Sucking decelerates the fluid particles through the porous wall hence reducing the concentration.

Concentration Profiles for Gr = -5

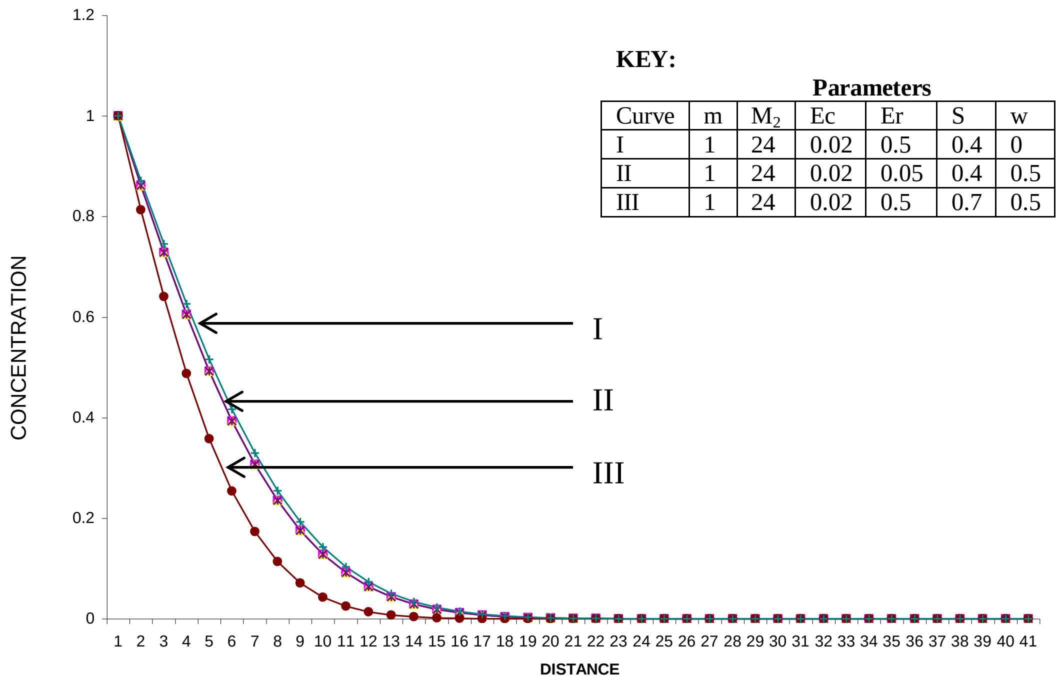


Fig 4.4: Concentration profiles in the presence of heating of the plate by free convection current for the case of Gr<0 (= -5)

Case 2

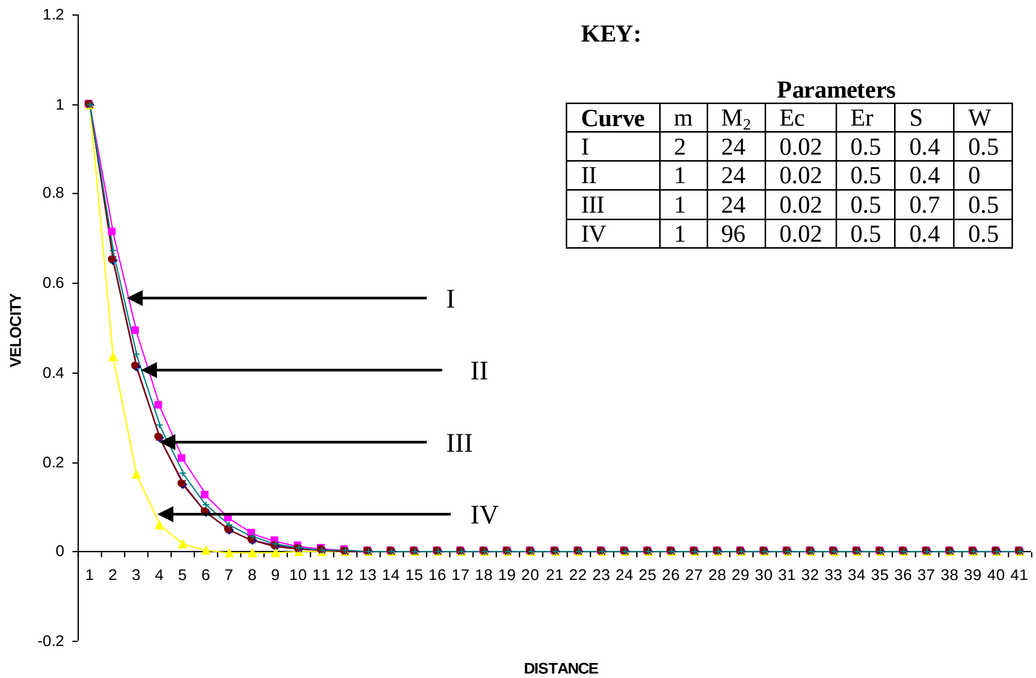
In the presence of cooling of the plate by free convectional currents i.e. when the Grashof number is greater than zero (equal to five) from Figure 4.5 and Figure 4.6 we note that:

- For Ec = 0.02, a decrease in the rotational parameter leads to an increase in both the primary and secondary velocity profiles. This is because the presence of the transverse magnetic field creates a resistive force similar to the drag force that acts in the opposite direction of the fluid; thus causing the velocity of the fluid to decrease.
- An increase in the Magnetic parameter leads to a decrease in the primary velocity and an increase in the secondary velocity profiles. Due the Lorentz force, there is a resistive force along the x-axis and this reduces the primary velocity but the secondary velocity profile increases since it is in the direction of the induced force.
- An increase in Hall parameter leads to an increase in the primary velocity and a decrease in the secondary velocity profiles. When the Hall parameter is increased the induced current along x-axis increases and this translates to an increase in the primary velocity while the induced current along the y-axis decreases slightly and thus a reduction in the secondary velocity profiles.
- We also observe that an increase in the suction parameter causes no effect in the primary velocity profiles but a decrease in secondary velocity profiles. Increasing suction parameter means pumping more fluid to the surface of the plate. This

does not affect the primary velocity profiles but decreases secondary velocity profiles since action and reaction forces which are in play are equal and opposite in nature.

- Removal of injection increases both the primary and secondary velocity profiles. This is because convectional currents which are interfering with the fluid flow are reduced and thereby increasing both primary and secondary velocity profiles.
- An increase in Eckert number results to an increase in both the primary and secondary velocity profiles. An increase in Eckert number means an increase in kinetic energy of the fluid particles and for this reason both primary and secondary velocity profiles.

Primary Velocity Profiles for Gr = 5



	Parameters					
Curve	m	M_2	Ec	Er	S	W
I	2	24	0.02	0.5	0.4	0.5
II	1	24	0.02	0.5	0.4	0
III	1	24	0.02	0.5	0.7	0.5
IV	1	96	0.02	0.5	0.4	0.5

Fig 4.5: Primary Velocity profiles in the presence of cooling of the plate by free convection current for the case of Gr>0 (= 5)

Secondary Velocity Profiles for Gr = 5

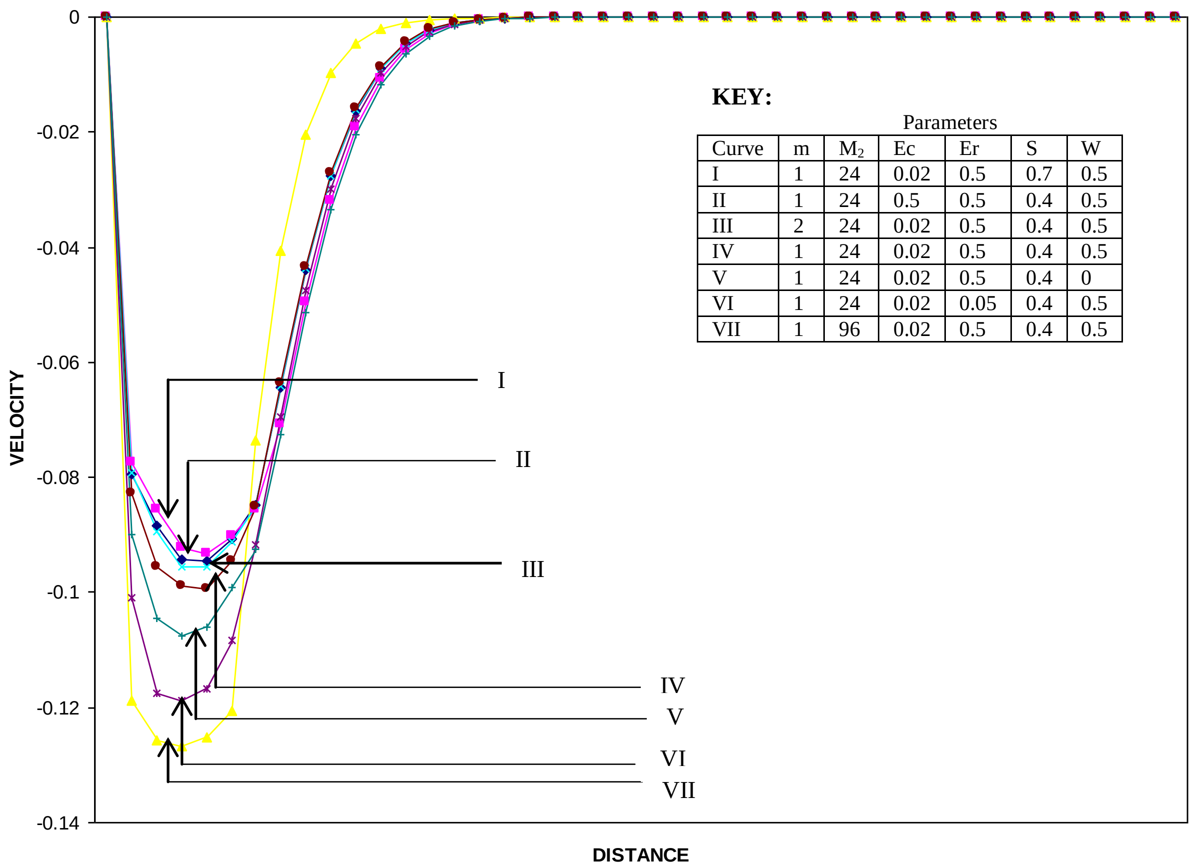
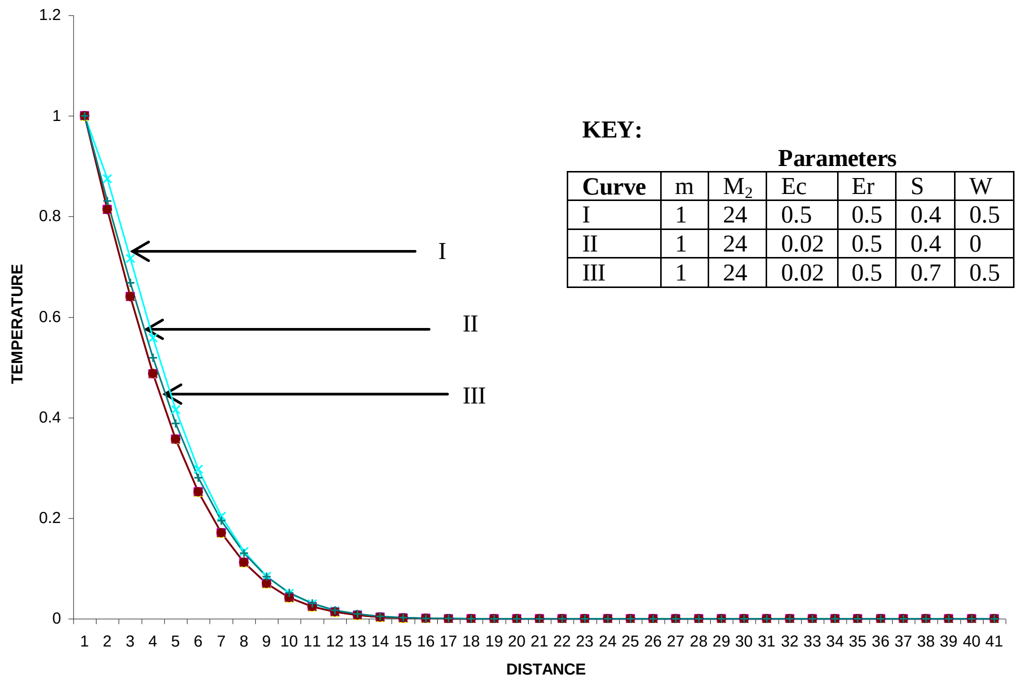


Fig 4.6: Secondary Velocity Profiles in the presence of cooling of the plate by free convection current for the case of Gr>0 (= 5)

From fig 4.7: Temperature profiles for Gr>0 we observe that:

- A decrease in the Rotational parameter Er, leads to an increase in the temperature profiles. The rotation causes the circulation of induced currents at the surface of the fluid, that is, the increase of the temperature affects the current distribution. Rotation leads up to additional transport; this contribution is a consequence of the decrease of the ion rotation. Viscous dissipation would immediately lead to an increase of ion-temperature, thus increasing ion momentum and thermal transport.
- An increase in Hall parameter causes an increase in the temperature profile. This is because in Hall parameter means an increase of ion collisions which translates to more thermal generation hence increasing the temperature profiles.
- Removal of injection causes a rise in temperature profiles. The high injection current causes a strong self-heating effect which reduces the quantum of the particles.
- Increase in Eckert number leads to an increase in the temperature profiles. Increasing the Eckert number causes the fluid to become warmer and therefore increase its temperature. This is attributed to the viscous dissipation. Increasing Ec can lead to a situation that the viscous dissipation becomes significant hence increasing the temperature.
- Increase in suction parameter has no effect on the temperature profiles. Increase in suction means a decrease in molecular diffusivity (D) and this means that the temperature profiles are not altered.

Temperature Profiles for Gr = 5



Curve	m	M_2	Ec	Er	S	W
I	1	24	0.5	0.5	0.4	0.5
II	1	24	0.02	0.5	0.4	0
III	1	24	0.02	0.5	0.7	0.5

Fig 4.7: Temperature profiles in the presence of cooling of the plate by free convection current for the case of Gr>0 (= 5)

From fig 4.8: Concentration profile for Gr > 0 we observe that;

- A decrease in the Rotational parameter Er has no effect to the concentration profiles. Rotation has been achieved by a transfer of angular momentum. Once this is drastically reduced, the rate at which the particles move and collide is too small such that the change is insignificant.
- An increase in Hall parameter has no effect to the concentration profile. An increase in Hall parameter which is due to the increase of collisions has no effect to the concentration profile. This is because there is no change in the charge carriers hence the effect is neutralized. Since no polarization voltage is imposed on the fluid, the concentration profile is not affected.
- Removal of injection causes a rise in concentration profiles. Removal of injection means an increase in the molecular diffusivity which consequently results in the rise of the concentration.
- Increase in the Magnetic parameter has no effect to concentration profiles. As M increases, the Lorentz force which tends to oppose the flow also increases. The effect is enhanced deceleration which when combined with the momentum diffusivity has no effect to concentration. Again there is no increment in the buoyancy ratio hence no change in the concentration.
- Increase in suction parameter leads to a decrease in the concentration profiles. Suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth. Sucking decelerates the fluid particles through the porous wall hence reducing the concentration.

Concentration Profiles for Gr = 5

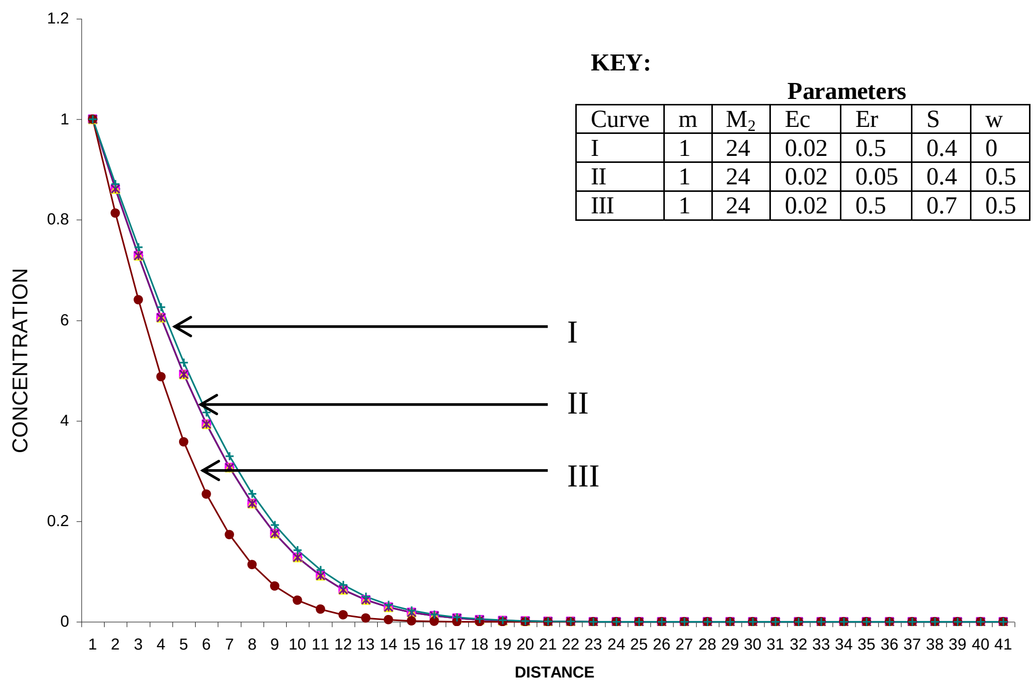


Fig 4.8 Concentration profiles of cooling of the plate by free convection current for the case of Gr>0 (= 5)

From Table 1 we observe that with Gr=5.0

- Removal of injection leads to a decrease in the rate of heat transfer.

  Removal of injection means the reduction of the particles which were causing collisions resulting in heat changes. This means that the collision times, allows the removal of air and the consequence is reduced heat transfer.

- An increase in suction parameter or magnetic parameter leads to an increase in the rate of heat transfer. The increase of S means a decrease of molecular diffusivity (D) that result in increment of the rate of heat transfer. The application of the externally variable magnetic field reduces the velocity vectors and leads to an increase in the rate of heat transfer.

- A decrease in rotational parameter Er leads to an increase in the rate of heat transfer. Due the presence of the Lorentz force and the gravitational force rotating at very low speeds, a friction factor is realized that results in thermal dispersion thereby increasing the rate of heat transfer.

- Removal of w decreases the rate of heat transfer. Injection causes convectional currents on the plate and this reduces the rate of heat transfer.

- Increase in the rotational parameter m leads to a decrease in the heat transfer. This reduction is due to the increase in the momentum, thermal and magnetic boundary layer thickness which in turn are caused by the deceleration of the magnetic field.

Table 1: Rate of heat transfer Nu with cooling Pr=0.71, Gr=5.0

m	M_2	Ec	Er	S	w	Nu
1	24	0.02	0.5	0.4	0.5	2.661606153
2	24	0.02	0.5	0.4	0.5	2.659182396
1	96	0.02	0.5	0.4	0.5	2.67362258
1	24	0.5	0.5	0.4	0.5	2.478347258
1	24	0.02	0.05	0.4	0.5	2.661558481
1	24	0.02	0.5	0.7	0.5	2.661558481
1	24	0.02	0.5	0.4	0	2.531911988

From Table 2 it is observed that with Gr=5.0

- Removal of injection parameter leads to an increase in (τ_x) but a decrease in (τ_y). The removal of injection parameter means reduction of the particles which were causing collisions this increases skin friction along x-axis and a decrease in skin friction along y-axis.
- An increase in Hall parameter leads to a decrease in (τ_x) but an increase in (τ_y). The skin friction in the y-direction tends to be negative since it is in the opposite direction to that of gravitational force.
- A decrease in rotational parameter Er lead to an increase in both (τ_x) and (τ_y). Due the presence of the Lorentz force and the gravitational force rotating at very low speeds, a friction factor is realized and hence an increase in both (τ_x) and (τ_y).
- Increase in Magnetic parameter leads to a rise in both (τ_x) and (τ_y). The application of the externally variable magnetic field reduces the velocity vectors

and since velocity is inversely proportional to frictional force and this means that both (τ_x) and (τ_y) increases.

- Increase in S leads to decrease in both (τ_x) and (τ_y). Basically suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth. Sucking decelerates the fluid particles through the porous wall, consequently this leads to decrease in both (τ_x) and (τ_y).

Table 2: Skin frictions with cooling Pr=0.71, Gr=5.0

m	M_2	Ec	Er	S	w	τ_x	τ_y
1	24	0.02	0.5	0.4	0.5	3.303558315	0.07671002
2	24	0.02	0.5	0.4	0.5	3.224548046	0.104524876
1	96	0.02	0.5	0.4	0.5	3.388281036	0.322132187
1	24	0.5	0.5	0.4	0.5	3.300116591	0.077290887
1	24	0.02	0.05	0.4	0.5	3.303796334	0.082941388
1	24	0.02	0.5	0.7	0.5	3.311764194	0.07528562
1	24	0.02	0.5	0.4	0	3.223194216	0.121241577

From table 3 (Gr=-5.0) we observe that

- Removal of injection causes a decrease in rate of heat transfer. Injection causes convectional currents on the plate and this reduces the rate of heat transfer.
- Increase in the Hall parameter lead to a decrease in rate of heat transfer. The magnetic field gives rise to a resistive force and slows down the movement of the fluid.
- A decrease in the Rotational parameter Er leads to a decrease in the rate of heat transfer. Rotation has been achieved by a transfer of angular momentum. Once this is drastically reduced, the rate at which the particles move and collide is too small thus the rate of heat transfer decreases.

- An increase in Eckert number Ec leads to a decrease in the heat transfer. Increase in Eckert number leads to increase in viscous dissipation and as a consequence, the rates of heat transfer decreases.
- Increase in the Suction parameter leads to a decrease in the rate of heat transfer. Since the plate is free convectively heated, increasing suction means decreasing molecular diffusivity which results to a decrease of the rate of heat transfer.
- An increase in Magnetic parameter leads to an increase in the rate of heat transfer. The application of the externally variable magnetic field reduces the velocity vectors and leads to an increase in the rate of heat transfer in a free convectional heating.

Table 3: Rate of heat transfer Nu with heating Pr=0.71, Gr=-5.0

m	M_2	Ec	Er	S	w	Nu
1	24	0.02	0.5	0.4	0.5	2.661961166
2	24	0.02	0.5	0.4	0.5	2.659542075
1	96	0.02	0.5	0.4	0.5	2.674019269
1	24	0.5	0.5	0.4	0.5	2.48783689
1	24	0.02	0.05	0.4	0.5	2.661914493
1	24	0.02	0.5	0.7	0.5	2.661914493
1	24	0.02	0.5	0.4	0	2.532264669

From Table 4 we observe that with Gr= -5.0

- Removal of injection or increase in Ec leads to an increase in both (τ_x) and (τ_y) . The removal of injection parameter means reduction of the particles which were causing collisions this increases skin friction along both x-axis and y-axis.

- An increase in Hall parameter leads to a decrease in (τ_x) but an increase in (τ_y). The skin friction in the y-direction tends to be negative since it is in the opposite direction to that of gravitational force.
- A decrease in Rotational parameter Er leads to an increase in both (τ_x) and (τ_y). Due the presence of the Lorentz force and the gravitational force rotating at very low speeds, a friction factor is realized and hence an increase in both (τ_x) and (τ_y).
- Increase in magnetic parameter leads to a rise in (τ_x) but a decrease (τ_y).

Hall currents due to the magnetic field give rise to a cross flow making the flow posses a resistive force that increases in the x- axis and decreases in the y-axis. It is observed that the primary effect of the magnetic field is to decelerate the flow.

- Increase in Suction leads to an increase in (τ_x) but a decrease in (τ_y).

increasing suction means pumping more fluid to the surface of the plate. This means that the convectional currents are increased and as a consequence it increases the skin friction along the x-axis and decreases the skin friction along the y-axis due the action against the gravitational force.

Table 4: Skin frictions with heating Pr=0.71, Gr=-5.0

m	M_2	Ec	Er	S	w	τ_x	τ_y
1	24	0.02	0.5	0.4	0.5	3.324194444	0.071301668
2	24	0.02	0.5	0.4	0.5	3.249333774	0.099589728
1	96	0.02	0.5	0.4	0.5	3.393047856	-0.3369896
1	24	0.5	0.5	0.4	0.5	3.327540829	0.070710633
1	24	0.02	0.05	0.4	0.5	3.3245373	0.077084675
1	24	0.02	0.5	0.7	0.5	3.332400323	0.069874435
1	24	0.02	0.5	0.4	0	3.24851838	0.115209036

CHAPTER FOUR

CONCLUSION AND RECOMMENDATIONS

4.0 INTRODUCTION

In this chapter, a conclusion on the results obtained in chapter four is presented and further areas of research recommended.

4.1 CONCLUSIONS

This study involved MHD free convection flow of a heat generating electrically conducting fluid past a semi-infinite vertical porous plate with variable suction. The effects of suction, injection, rotation and Hall current in presence of variable magnetic filed considering both free convectional heating and cooling were studied. The system was set in a rotational motion and a strong variable magnetic field applied transversely. One objective of this study was to investigate the effects of various fluid flow parameters on the velocity, concentration and temperature profiles. The next objective was to study how these parameters affect skin friction, mass transfer and the rate of heat transfer. This study came up with the fluid flow model that assisted in modeling the equations governing the flow problem. Equations governing the fluid flow that were developed are momentum equation, energy equation, concentration equation and induction equation. A non-dimensionalization scheme was used to non-dimensionalize the governing equations. A finite difference scheme was used to write the equations in finite difference form. A computer program was written and run to generate velocity, temperature and concentration profiles and presented in graphical form as depicted in chapter four. Skin friction and rate of heat transfer were computed and presented in tabular form.

The following conclusions on the effects of each non-dimensional parameter were made;

Increase of suction parameter: There was no effect on primary velocity profiles, temperature profiles in both free convectional heating and cooling of the plate, though there was a decrease in; secondary velocity profiles, concentration profiles, rate of heat transfer and skin friction along y-axis. Free convectional heating of the plate increased skin friction along x-axis.

Removal of injection: Both free convectional heating and cooling of the plate increased primary and secondary velocity profiles, concentration profiles, temperature profiles and skin friction along x-axis, but there was a decrease in the rate of heat transfer and skin friction along y-axis.

Increase in Eckert number: In both free convectional heating and cooling of the plate, there was an increase in primary and secondary velocity profiles and temperature profiles, but a decrease in heat transfer on heating of the plate. In both cases, there was no effect on concentration profiles.

Increase of magnetic parameter: An increase of magnetic parameter free convectional heating and cooling of the plate increased secondary velocity profiles, rate of heat transfer and skin friction both along x-axis and y-axis, but there was no effect on concentration profiles. The free convectional heating of the plate increased primary velocity profiles but decreased primary velocity profiles on free convectional cooling of the plate.

Decrease of rotational parameter: A decrease of rotational parameter in both free convectional heating and cooling of the plate increased primary and secondary velocity profiles, temperature profiles and both skin friction along x-axis and y-axis, but there was no effect on concentration profiles. The rate of heat transfer decreased on convectional heating of the plate while it increased on convectional cooling of the plate.

Increase of hall parameter: Increase of hall parameter both on free convectional cooling and heating increased skin friction along y-axis and decreased skin friction along x-axis but there was no effect on concentration. It was noted that there was a decrease in temperature profiles and the rate of heat transfer in free convectional heating on the plate and an increase of primary velocity profiles and temperature profiles in free convectional cooling on the plate.

4.2 RECOMMENDATIONS

This study considered magnetohydrodynamic free convection of a heat generating fluid past a semi-infinite vertical porous plate with variable suction. The study on MHD flow for a vertical semi-infinite plate in a dissipative rotating fluid is a wide area of study. Most of it was not considered in this thesis and can be investigated in future because of its importance in its applications in Engineering.

Some of the areas that can be researched on in future include:

1. Fluid flow that is compressible
2. Fluid flow that is rotational

3. Fluid flow past a vertical semi-infinite plate with Hall and Ion-slip current
4. Fluid flow that is compressible past an infinite vertical porous plate with Hall and Ion-slip current
5. Fluid flow that is compressible and rotational past an semi-infinite vertical porous plate with variable suction
6. Fluid flow that is compressible and rotational past an infinite vertical porous plate with variable suction.

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